Lecture (1)

# CIRCUIT THEORY

## Introduction

Circuit theory is an important and perhaps the oldest branch of electrical engineering. A circuit is an interconnection of electrical elements. These include **passive elements**, such as resistances, capacitances, and inductances, as well as **active elements** such as sources. Two variables, namely voltage and current variables are associated with each circuit element. There are two aspects to circuit theory: analysis and design. Circuit analysis involves the determination of current and voltage values in different elements of the circuit, given the values of the sources. On the other hand, circuit design focuses on the design of circuits that exhibit a certain prespecified voltage or current characteristics at one or more parts of the circuit.

## System of Units

1) The English system.
2) The metric system which is subdivided into two interrelated standards:
    - The **MKS** system uses **M**eters, **K**ilograms, and **S**econds.
    - The **CGS** system uses **C**entimeters, **G**rams, and **S**econds.
3) The international metric system of units (SI).

The international system of units, commonly called SI, is used in electricity. The seven base units of SI are listed in Table (1.1). The two supplementary units of SI are plane angle and solid angle Table (1.2).

Table (1.1): Base Units of the International Metric System

| Quantity | Base Unit | Symbol |
|---|---|---|
| Length | meter | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Light intensity | candela | cd |
| Amount of substance | mole | mol |

Table (1.2): Supplementary SI Units

| Quantity | Unit | Symbol |
|---|---|---|
| Plane angle | radian | rad |
| Solid angle | steradian | sr |

Other common units can be derived from the base and supplementary units. For example, the unit of charge is the coulomb, which is derived from the base units of second and ampere. Most of the units that are used in electricity are derived ones Table (1.3).

Table (1.3): Derived SI Units

| Quantity | Unit | Symbol |
|---|---|---|
| Energy | joule | J |
| Force | newton | N |
| Power | watt | W |
| Electric charge | coulomb | C |
| Electric potential | volt | V |
| Electric resistance | ohm | Ω |
| Electric conductance | siemens | S |
| Electric capacitance | farad | F |
| Electric inductance | henry | H |
| Frequency | hertz | Hz |
| Magnetic flux | weber | wb |
| Magnetic flux density | tesla | T |

## Metric Prefixes

In the study of basic electricity, some electrical units are too small or too large to express conveniently. For example, in the case of resistance, we often use values in thousands or millions of ohms (Ω). The prefix kilo (denoted by the letter k) is a convenient way of expressing a thousand. Thus, instead of saying a resistor has a value of 10 000 Ω, we normally refer to it as a 10-kilohm (10-kΩ) resistor. In the case of current, we use expressions such as milli amperes and microamperes. The prefix milli is a short way of saying a thousandth and micro is a short way of saying a millionth. Thus 0.012 A becomes 12 milliamperes (mA) and 0.000 005 A becomes 5 microamperes ($\mu$A). Table (1.4) lists the metric prefixes commonly used in electricity and their numerical equivalents.

Table **(1.4): SI unit prefixes**

| Multiplier | Prefix | Symbol |
|---|---|---|
| $10^{18}$ | exa | E |
| $10^{15}$ | peta | P |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{2}$ | hecto | h |
| 10 | deka | da |
| $10^{-1}$ | deci | d |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | µ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

## Definitions & Terminologies

1- **Electric charge (Q)** : is a physical property of electrons and protons in the atoms that gives rise to force between atoms. The charge is measured in coulomb [C].

The **coulomb** is defined as the quantity of electricity which flows in an electric circuit when a current of one ampere flow for one second.

The charge of proton is arbitrarily chosen as positive and has the value of ($1.6 \times 10^{-19}$ C), whereas the charge of an electron is chosen as negative with a value of ($-1.6 \times 10^{-19}$ C).

**Note** that, like charges repel while unlike charges attract each other.

$$Q = I.t \qquad [C]$$

Where, Q:- electric chargein coulomb.
I:- electric current in ampere.
t:- time in second.

The total charge (Q) transferred during the time from ($t_1$) to ($t_2$) can be calculated as,

$$Q = \int_{t_1}^{t_2} i.dt \qquad [C]$$

**Example 1:** If a current of 5A flows for 2 minutes. Find the quantity of electricity transferred.

**Solution:**

Quantity of electricity is (Q) =I.t

I=5A, t = 2 × 60 = 120 second [S]

Hence Q = 5 × 120 = 600 C

2- **Current (I) :** is the flow rate of electric charge that is measured as coulombs per second. The unit of electric current is ampere [A].

An ampere is defined as the flow of charge at the rate of one coulomb per second (1A = 1C/S).

$$I = \frac{Q}{t} \quad [A] \qquad \text{or} \qquad i(t) = \frac{dq}{dt} \quad [A]$$

**Note:** the conventional direction of current flow is reverse to the direction of electrons flow as shown in **Figure (1.1)**.

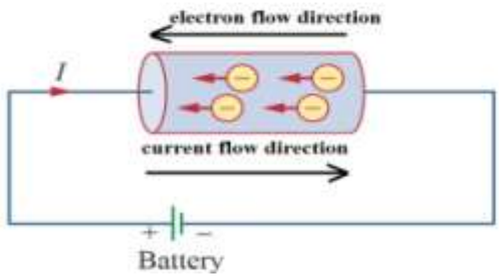


Figure (1.1)

3- **Direct current (dc):** is the current that remains constant in direction & magnitude (constant with time) as shown in **Figure (1.2)(a)**.The symbol (I) is usually used to represent such a constant current. The reason for such unidirectional current is that the voltage source maintain the same polarity for the output voltage.
The voltage supplied by these sources is called direct current voltage or dc voltage.

4- **Alternating current (ac):** is the current that reverse or alternates in polarity with time such sinusoidal current as shown in **Figure (1.2)(b)**, ac usually represent by a symbol *i* or *i(t)*.

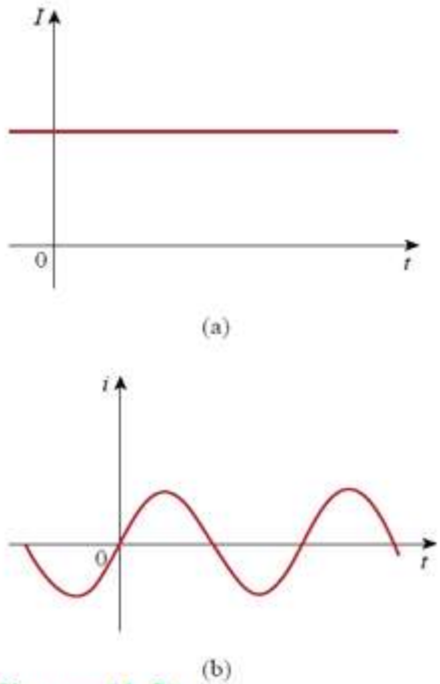



**Figure (1.2)**
Two common types of current: (a) direct current (dc), (b) alternating current (ac).

5- **Voltage or Potential Difference (PD):-** potential difference ($V_{AB}$) between two points A & B, is the amount of energy required (or work done) to move a unit positive charge from point A to point B. If this energy is positive, then ($V_{AB}$) is positive & point A is higher potential with respect to (w.r.t.) point B.

$$V_{AB} = V_A - V_B \qquad \& \qquad V_A > V_B$$

The voltage is measured using the unit of volt [V], which is the work of 1-joule used to move a charge of 1-coulombbetween two points.

$$V = \frac{W}{Q} \qquad \text{or} \qquad v = \frac{dw}{dq}$$

Where, W:- is the energy or work done.

6- **Power (P):-** it is defined as the rate of work done, or it is the rate of energy with respect to time.

$$P = \frac{W}{t} \qquad [\ \text{watt} = \frac{\text{joule}}{\text{second}}\ , \text{or} \quad W = \frac{J}{S}\ ]$$

$$\text{Or} \quad P = \frac{W}{t} \quad \text{or} \quad P = \frac{QV}{t} \quad \text{or} \quad P = IV$$

**Note:-**

- 1 watt = $\frac{1\text{joule}}{\text{second}}$
- 1 horse power (h.p.) = 746 watt
- Negative power means the element gives energy to network (such as sources).
- Positive power means the element absorbs energy from network (such as resistors).

7- **Energy (W) :-** It is quantity represents the product of power (P) & the period (t).

$W = P.t$ [watt.second or w.s or joule]

$W = Q.V$ Or $W = \int_{t_1}^{t_2} P.dt$

$$\text{Energy [kW.h]} = \frac{P[w] \times t[s]}{1000}$$

**Example 2:** How much energy does a 100w electric bulb consumes in 2 hours?

**Solution:**

$W = P.t = 100 \times 2 = 200$ w.h

Or

$W = P.t = 100 \times (2 \times 60 \times 60) = 720000$ J = 720 KJ

**Example 3:** Determine the energy expended in moving a charge of 50 μC through a potential difference of 6V.

**Solution:**

$W = Q.V = (50 \times 10^{-6}) \times 6 = 300 \times 10^{-6}$ J = 300 μ J

**Example 4:** A voltage source of 5V, supplies a current of 3A for 10 minutes. How much energy is supplied in this time?

**Solution:**

W =P.t = (V.I)t

$= (5 \times 3) \times (10 \times 60)$

= 9000 [w.s or joule] = 9 KJ

8- **Circuit Elements** :- can be divided into two types:

(i) *Active Elements*: the elements which are able to supply energy to the network such as voltage & current sources which includes generators, batteries & operational amplifier.

(ii) *Passive Elements*: the elements which take energy from sources & either convert it to another form, or store it as electric or magnetic field, such as resistors, inductors & capacitors.

**Note:** In passive elements the polarity of voltage is such that current flow from positive terminal to negative terminal (reverse to the direction of current in voltage source), as shown in **Figure (1.3)**

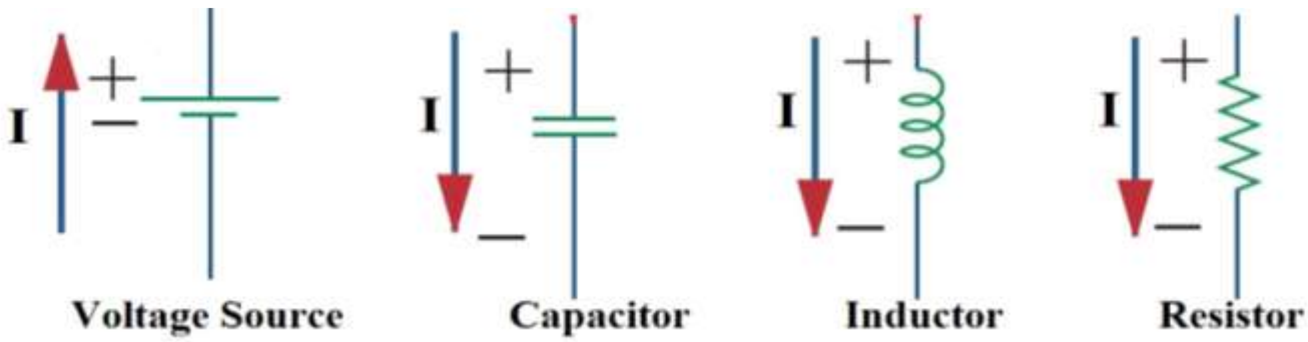



**Figure (1.3)**

**Sources:** sources (weather voltage or current sources) can be divided into two types,

(i) Independent Sources. (ii) Dependent Sources.

(i) ***Independent Source***, is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

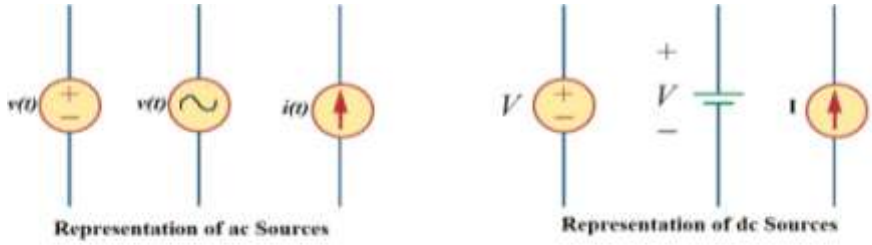
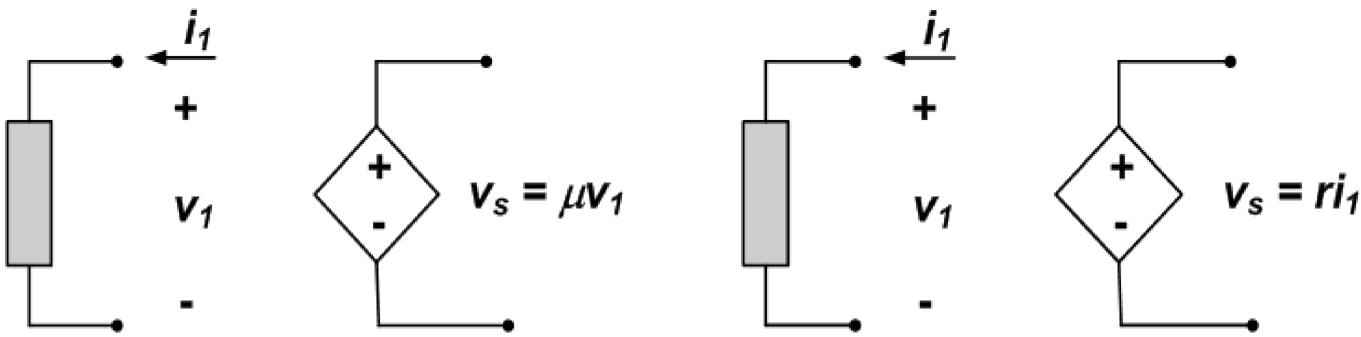


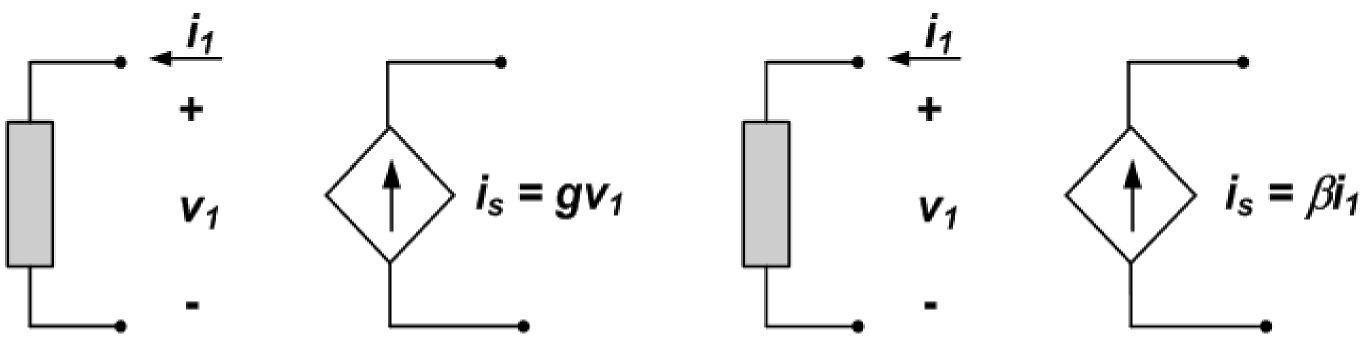

**Figure (1.4):** Representation of sources

(ii) ***Dependent (or controlled) Source***, an active element in which the source quantity is controlled by another voltage or current. It can be classified as ,

1) A voltage-controlled voltage source (VCVS).
2) A current-controlled voltage source (CCVS).
3) A voltage-controlled current source (VCCS).
4) A current-controlled current source (CCCS).




(a)VCVS, μ is voltage gain (dimensionless) (b)CCVS, r is transresistance (Ω)




(c)VCCS, g is transconductance (siemens) (d)CCCS, β is current gain (dimensionless)

**Figure (1.5):** Representation of controlled sources

# Resistance

The flow of charge through any material encounters an opposing force similar in many respects to mechanical friction. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat*, is called the **resistance** of the material. The unit of measurement of resistance is the **ohm,** for which the symbol is (Ω) the capital Greek letter omega.
The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

**1. Material** (it depends on the nature of the material).
**2. Length** (it varies directly as it's length).
**3. Cross-sectional area** (it varies inversely as the cross-section of the conductor).
**4. Temperature** (it depends on the temperature of the conductor).

As the temperature of most conductors increases, the increased motion of the particles within the molecular structure makes it increasingly difficult for the "free" carriers to pass through, and the resistance level increases.
At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by

$$R \propto \frac{l}{A} \quad (\Omega)$$

$$\boxed{R = \rho \frac{l}{A}} \quad (\Omega)$$

$$\boxed{\rho = R \frac{A}{l}} \quad (\Omega.m)$$

Where ρ (Greek letter rho) is a characteristic of the material called the **specific resistance** or **resistivity,** *l* is the length of the material, and *A* is the cross-sectional area of the material.

Table (1.5): Resistivity of Materials

| Material | Resistivity, $\rho$, at 20°C (Ω-m) |
|---|---|
| Silver | $1.645 \times 10^{-8}$ |
| Copper | $1.723 \times 10^{-8}$ |
| Gold | $2.443 \times 10^{-8}$ |
| Aluminum | $2.825 \times 10^{-8}$ |
| Tungsten | $5.485 \times 10^{-8}$ |
| Iron | $12.30 \times 10^{-8}$ |
| Lead | $22 \times 10^{-8}$ |
| Mercury | $95.8 \times 10^{-8}$ |
| Nichrome | $99.72 \times 10^{-8}$ |
| Carbon | $3500 \times 10^{-8}$ |
| Germanium | 20–2300* |
| Silicon | ≅500* |
| Wood | $10^{8}$–$10^{14}$ |
| Glass | $10^{10}$–$10^{14}$ |
| Mica | $10^{11}$–$10^{15}$ |
| Hard rubber | $10^{13}$–$10^{16}$ |
| Amber | $5 \times 10^{14}$ |
| Sulphur | $1 \times 10^{15}$ |
| Teflon | $1 \times 10^{16}$ |

## Conductance

By finding the reciprocal of the resistance of a material, we have a measure of how well the material will conduct electricity. The quantity is called conductance, has the symbol G, and is measured in Siemens (S) or (moh). In equation form, conductance is,

$$G = \frac{1}{R} \qquad \text{(siemens, S)}$$

$$G = \frac{A}{\rho l}$$

$$G = \frac{\gamma A}{l} \qquad \text{(S)}$$

Where $\gamma = \frac{1}{\rho}$ ( called conductivity of material in [s/m])

A resistance of 1 MΩ is equivalent to a conductance of $10^{-6}$ S, and a resistance of 10 Ω is equivalent to a conductance of $10^{-1}$ S. The larger the conductance, therefore, the less the resistance and the greater the conductivity.

**Example 5:** What is the relative increase or decrease in conductivity of a conductor if the area is reduced by 30% and the length is increased by 40%? The resistivity is fixed.

**Solution:**

$$G_i = \frac{A_i}{\rho_i l_i}$$

with the subscript i for the initial value. Using the subscript n for new value:

$$G_1 = \frac{A_1}{\rho_1 l_1} \qquad \ldots(1)$$

$$G_2 = \frac{A_2}{\rho_2 l_2} \qquad \ldots(2)$$

$$A_2 = 0.7\,A_1 \qquad \ldots(3)$$

$$l_2 = 1.4\,l_1 \qquad \ldots(4)$$

Substitute (3) & (4) in equation (2).

$$G_2 = \frac{0.7A_1}{\rho_1 \times 1.4 l_1} = \frac{0.7}{1.4} \times \frac{A_1}{\rho_1 l_1} = 0.5 \times \frac{A_1}{\rho_1 l_1} = 0.5\,G_1$$

**Example 6:** Most homes use solid copper wire having a diameter of 1.63 mm to provide electrical distribution to outlets and light sockets. Determine the resistance of 75 meters of a solid copper wire having the above diameter.

**Solution:**

We will first calculate the cross-sectional area of the wire.

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (1.63 \times 10^{-3})^2}{4} = 2.09 \times 10^{-6} \ m^2$$

$$R = \rho \frac{l}{A} = \frac{(1.723 \times 10^{-8} \ \Omega.m) \times (75 \ m)}{2.09 \times 10^{-6} \ m^2} = 0.619 \ \Omega$$

**Example 7:** Bus bars are bare solid conductors (usually rectangular) used to carry large currents within buildings such as power generating stations, telephone exchanges, and large factories. Given a piece of aluminum bus bar as shown in **Figure (1.6)**, determine the resistance between the ends of this bar at a temperature of 20°C.

**Solution:**

The cross-sectional area is

$A$ = (150 mm)(6 mm)
= (0.15 m)(0.006 m)
= $0.0009 \ m^2$
= $9.00 \times 10^{-4} \ m^2$

The resistance between the ends of the bus bar is determined as

$$R = \rho \frac{l}{A} = \frac{(2.825 \times 10^{-8} \ \Omega.m) \times (270 \ m)}{9 \times 10^{-4} \ m^2}$$

$= 8.48 \times 10^{-3} \ \Omega$
$= 8.48 \ m\Omega$

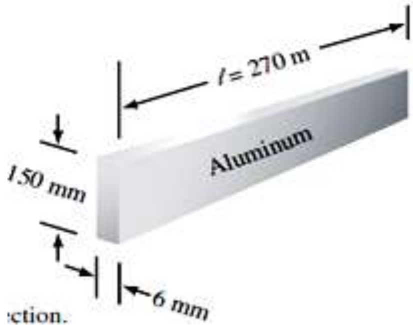



**Figure (1.6):** Conductor with rectangular cross section

**Example 8:** A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 $mm^2$. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ–m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.

**Solution:**

Length of the coil, $l = 0.8 \times 2000 = 1600$ m ;
$A = 0.8 \ mm^2 = 0.8 \times 10^{-6} \ m^2$.

$$R = \rho \frac{l}{A} = \frac{(0.02 \times 10^{-6} \ \Omega.m) \times (1600 \ m)}{0.8 \times 10^{-6} \ m^2}$$

= **40 Ω**

Power absorbed = $V^2 / R = 110^2/40$ = **302.5 W**

**Figure (1.7)**

**Example 9: (a)** A rectangular carbon block has dimensions 1.0 cm × 1.0 cm × 50 cm. **(i)** What is the resistance measured between the two square ends ? **(ii)** between two opposing rectangular faces / Resistivity of carbon at 20°C is $3.5 \times 10^{-5}$ Ω-m.
**(b)** A current of 5 A exists in a 10-Ω resistance for 4 minutes **(i)** how many coulombs and **(ii)** how many electrons pass through any section of the resistor in this time ? Charge of the electron = $1.6 \times 10^{-19}$ C.

**Solution:**

**(a) (i)**

$A = 1 \times 1 = 1\ cm^2 = 10^{-4}\ m^2$ ; $l = 0.5$ m

$$\therefore \quad R = R = \rho \frac{l}{A} = \frac{(3.5 \times 10^{-5}\ \Omega.m) \times (0.5\ m)}{10^{-4}\ \ m^2} = \mathbf{0.175\ \Omega}$$

**(ii)** $l = 1$ cm; $A = 1 \times 50 = 50\ cm^2 = 5 \times 10^{-3}\ m^2$

$$R = \frac{(3.5 \times 10^{-5}\ \Omega.m) \times (10^{-2}\ m)}{5 \times 10^{-3}\ \ m^2} = \mathbf{7 \times 10^{-5}\ \Omega}$$

**(b) (i)** $Q = It = 5 \times (4 \times 60) = \mathbf{1200\ C}$

**(ii)** $n = Q / e = 1200 / (1.6 \times 10^{-19}) = \mathbf{75 \times 10^{20}}$

**Example 10:** The resistivity of a ferric-chromium-aluminium alloy is $51 \times 10^{-8}$ Ω-m. A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between **(a)** opposite ends and **(b)** opposite sides.

**Solution:**

**(a)** As seen from **Figure (1.8)(a)** in this case,
$l = 15$ cm = 0.15 cm
$A = 6 \times 0.014 = 0.084\ cm^2 = 0.084 \times 10^{-4}\ m^2$

$$R = \frac{(51 \times 10^{-8}\ \Omega.m) \times (0.15\ m)}{0.084 \times 10^{-4}\ \ m^2}$$
$$= \mathbf{9.1 \times 10^{-3}\ \Omega}$$

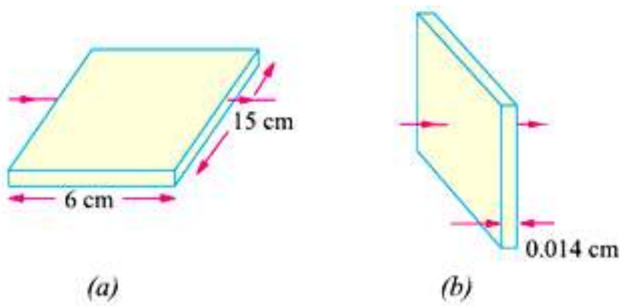



**Figure (1.8)**

**(b)** As seen from **Figure (1.8)(b)** here
$l = 0.014\ cm = 14 \times 10^{-5}$ m
$A = 15 \times 6 = 90\ cm^2 = 9 \times 10^{-3}\ m^2$

$$\therefore \quad R = \frac{(51 \times 10^{-8}\ \Omega.m) \times (14 \times 10^{-5} m)}{9 \times 10^{-3}\ \ m^2} = \mathbf{79.3 \times 10^{-10}\ \Omega}$$

## FIXED RESISTORS

A ***fixed resistor*** is one that has a single value of resistance which remains constant under normal conditions. The two main types of fixed resistors are carbon-composition and wire-wound resistors.

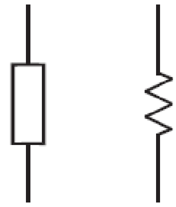


Figure (1.9)

## VARIABLE RESISTORS

Variable resistors are used to vary or change the amount of resistance in a circuit. Variable resistors are called **potentiometers** or **rheostats.**

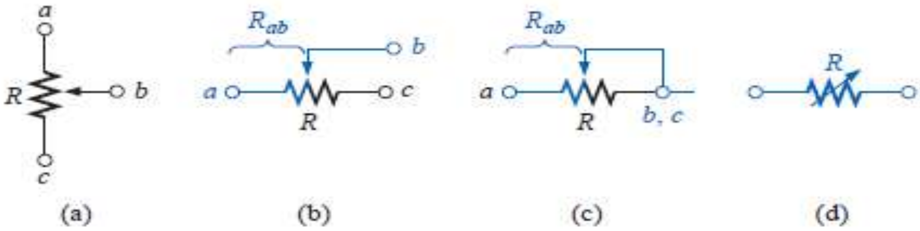



Figure (1.10)

## Color Coding of Resistors

Large resistors such as the wire-wound resistors or the ceramic-encased power resistors have their resistor values and tolerances printed on their cases. Smaller resistors, whether constructed of a molded carbon composition or a metal film, may be too small to have their values printed on the component. Instead, these smaller resistors are usually covered by an epoxy or similar insulating coating over which several colored bands are printed radially as shown in Figure below.

The colored bands provide a quickly recognizable code for determining the value of resistance, the tolerance (in percentage), and occasionally the expected reliability of the resistor. The colored bands are always read from left to right, left being defined as the side of the resistor with the band nearest to it.

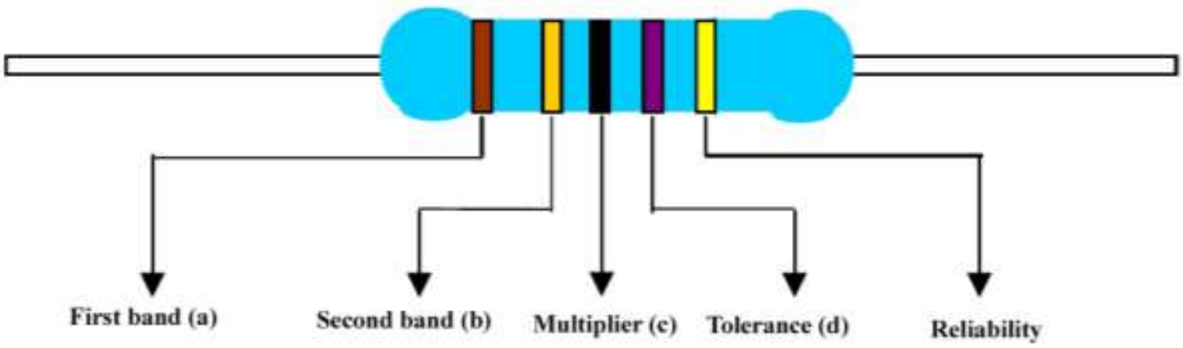



Figure (1.11): Color code resistor

The first two bands represent the first and second digits of the resistance value. The third band is called the multiplier band and represents the number of zeros following the first two digits; it is usually given as a power of ten.

The fourth band indicates the tolerance of the resistor, and the fifth band (if present) is an indication of the expected reliability of the component. The reliability is a statistical indication of the expected number of components which will no longer have the indicated resistance value after 1000 hours of use. For example, if a particular resistor has a reliability of 1% it is expected that after 1000 hours of use, no more than one resistor in 100 is likely to be outside the specified range of resistance as indicated in the first four bands of the color codes.

Color band resistor can be evaluated using the following relation,

$$\mathbf{R_{color} = ab \times 10^{c} \pm d}$$

Table (1.6) shows the colors of the various bands and the corresponding values.

Table (1.6): Resistor Color Code

| Color | Band 1 Sig. Fig. | Band 2 Sig. Fig. | Band 3 Multiplier | Band 4 Tolerance | Band 5 Reliability |
|---|---|---|---|---|---|
| Black | | 0 | $10^0 = 1$ | | |
| Brown | 1 | 1 | $10^1 = 10$ | | 1% |
| Red | 2 | 2 | $10^2 = 100$ | | 0.1% |
| Orange | 3 | 3 | $10^3 = 1\,000$ | | 0.01% |
| Yellow | 4 | 4 | $10^4 = 10\,000$ | | 0.001% |
| Green | 5 | 5 | $10^5 = 100\,000$ | | |
| Blue | 6 | 6 | $10^6 = 1\,000\,000$ | | |
| Violet | 7 | 7 | $10^7 = 10\,000\,000$ | | |
| Gray | 8 | 8 | | | |
| White | 9 | 9 | | | |
| Gold | | | 0.1 | 5% | |
| Silver | | | 0.01 | 10% | |
| No color | | | | 20% | |

**Example 11:** Determine the resistance of a carbon film resistor having the color codes shown in **Figure (1.12)**.

**Solution:**

From Table (1.6), we see that the resistor will have a value determined as,

$R_{color} = ab \times 10^{c} \pm d$

$= 18 \times 10^{3} \pm 5\%$

$= 18\ k\Omega \pm (0.05 \times 18\ k\Omega)$

$= 18\ k\Omega \pm (0.9\ k\Omega)$ with a reliability of 0.1%

$\therefore \quad R = 18 + 0.9 = 18.9\ k\Omega$

Or $R = 18 - 0.9 = 17.1\ k\Omega$

This specification indicates that the resistance will fall between 17.1 kΩ and 18.9 kΩ. After 1000 hours, we would expect that no more than 1 resistor in 1000 would fall outside the specified range.

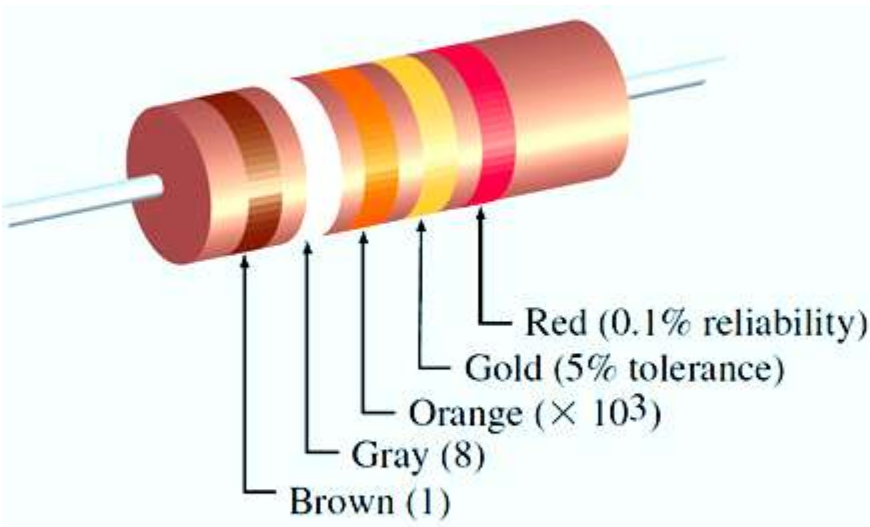



**Figure (1.12)**

**Lecture (2)**

# Ohm's law

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \qquad \ldots(1)$$

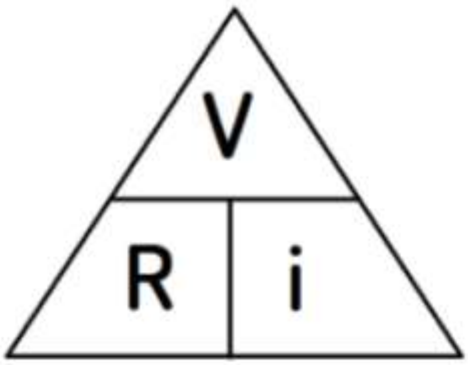
Ohm defined the constant of proportionality for a resistor to be the resistance, R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (1) becomes

$$v = iR \qquad \ldots(2)$$

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

$$V = iR$$

$$R = \frac{V}{i}$$




$$i = \frac{V}{R}$$

**Figure (2.1)**: Ohm's law triangle

**Example 1:** an electric conductor draws 2 A at 120 V. Find the resistance.

**Solution:**

From Ohm's law, $R = \frac{V}{i} = \frac{120}{2} = 60\ \Omega$

## Nodes, Branches, and Loops

A **branch** represents a single element such as a voltage source or a resistor.

A **node** is the point of connection between two or more branches.

A **loop** is any closed path in a circuit.

A network with (b) branches, (n) nodes, and ($l$) independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

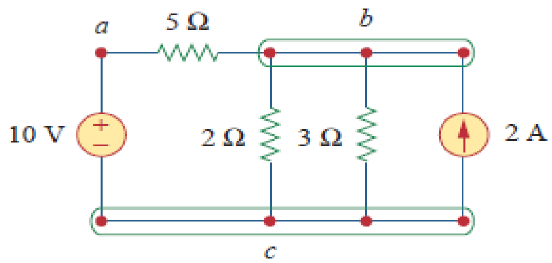



**Figure (2.2)(a)**: Nodes, branches and loops

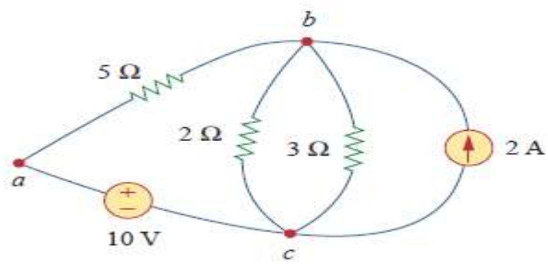



**Figure (2.2)(b)**: The three nodes circuit of **Figure (2.2)**(a) is redrawn.

**Example 2:** Determine the number of branches and nodes in the circuit shown below.

**Solution:**

Independent loops ($l$) = 2.
Number of nodes (n) = 3.
$b = l + n - 1 = 2+3-1 = 4$ branches

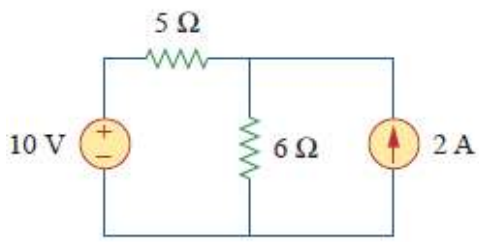


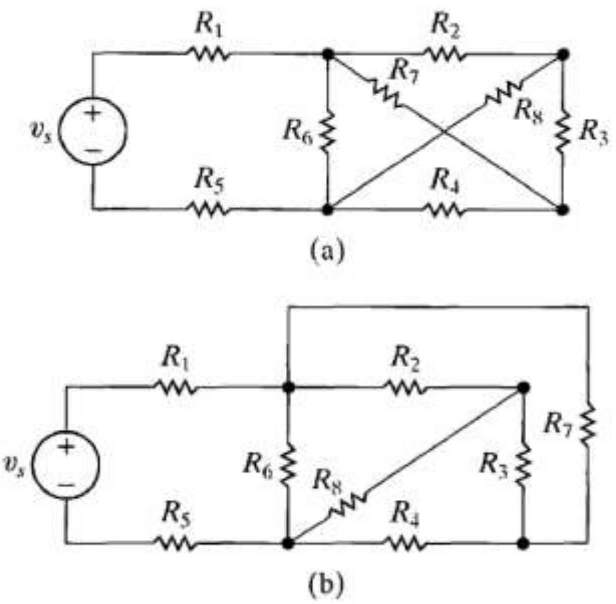

**Figure (2.3)**

## Planar & non-planar Circuits

**Planar circuits**—that is, those circuits that can be drawn on a plane with no crossing branches. A circuit that is drawn with crossing branches still is considered planar if it can be redrawn with no crossover branches. For example, the circuit shown in **Figure (2.4)** (a) can be redrawn as **Figure (2.4)** (b); the circuits are equivalent because all the node connections have been maintained. Therefore, **Figure (2.4)** (a) is a planar circuit because it can be redrawn as one.

**A non-planar circuit**—it cannot be redrawn in such a way that all the node connections are maintained and no branches overlap as shown in **Figure (2.5)**.

**Note:-** The node-voltage method is applicable to both planar and non-planar circuits, whereas the mesh-current method is limited to planar circuits.




(a)A planar circuit (b) The same circuit redrawn to verify that it is a planar

**Figure (2.4)**

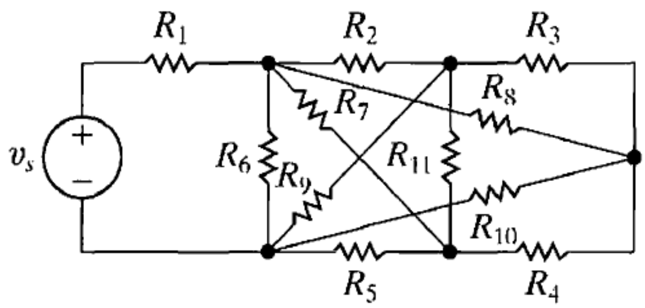


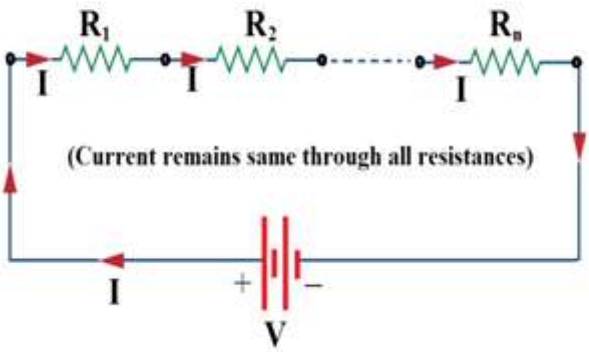
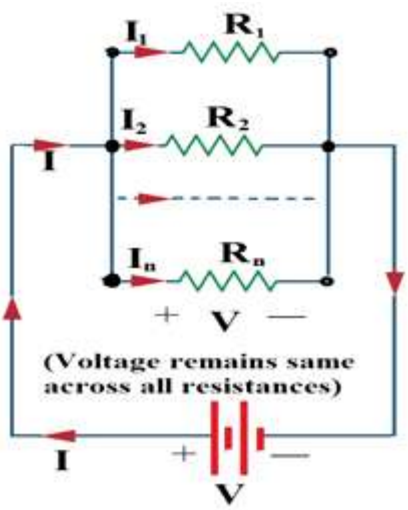

**Figure (2.5)**

## Series and Parallel Resistances

Two or more elements are in series if they exclusively share a single node and consequently carry the same current.

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

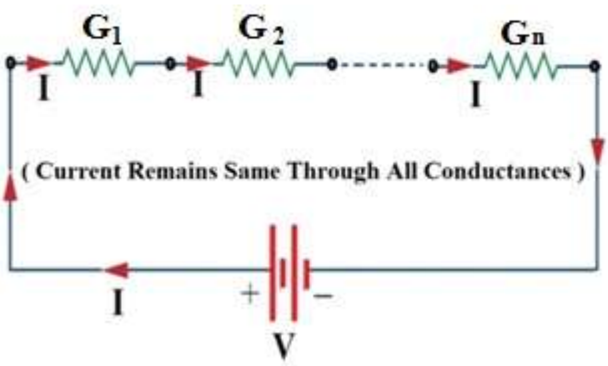
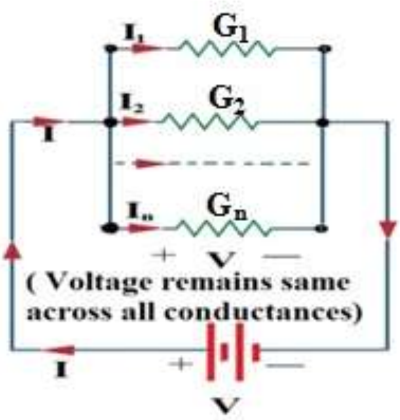
**Table (2.1)**

| NO. | Series Circuit | Parallel Circuit |
|---|---|---|
| 1 | The connection is as shown   | The connection is as shown   |
| 2 | The same current flows through each resistance. $I = I_1 = I_2 = \ldots = I_n$ | The same voltage exists across all the resistances in parallel. $V = V_1 = V_2 = \ldots = V_n$ |
| 3 | The voltage across each resistance is different. | The current through each resistance is different. |
| 4 | The sum of the voltages across all the resistances is the supply voltage. $V = V_1 + V_2 + \ldots + V_n$ | The sum of the currents through all the resistances is the supply current. $I = I_1 + I_2 + \ldots + I_n$ |
| 5 | The equivalent resistance is, $R_{eq} = R_1 + R_2 + \ldots + R_n$ | The equivalent resistance is, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$ |
| 6 | The equivalent resistance is the largest than each of the resistances in series. $R_{eq} > R_1, R_{eq} > R_2 \ldots, R_{eq} > R_n$ | The equivalent resistance is the smaller than the smallest of all the resistances in parallel. |

## Series and Parallel Conductances

$$G_{eq} = \frac{1}{R_{eq}},\ G_1 = \frac{1}{R_1},\ G_2 = \frac{1}{R_2},\ \ldots,\ G_n = \frac{1}{R_n}$$

**Table (2.2)**

| NO. | Series Circuit | Parallel Circuit |
|---|---|---|
| 1 | The connection is as shown<br><br> | The connection is as shown<br><br> |
| 2 | The same current flows through each conductance.<br>$I = I_1 = I_2 = \ldots = I_n$ | The same voltage exists across all the conductances in parallel.<br>$V = V_1 = V_2 = \ldots = V_n$ |
| 3 | The voltage across each conductance is different. | The current through each conductance is different. |
| 4 | The sum of the voltages across all the conductances is the supply voltage.<br>$V = V_1 + V_2 + \ldots + V_n$ | The sum of the currents through all the conductances is the supply current.<br>$I = I_1 + I_2 + \ldots + I_n$ |
| 5 | The equivalent conductance is,<br>$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \ldots + \frac{1}{G_n}$ | The equivalent conductance is,<br>$G_{eq} = G_1 + G_2 + \ldots + G_n$ |
| 6 | The equivalent conductance is the smaller than the smallest of all the conductances in parallel. | The equivalent conductance is the largest than each of the conductances in series.<br>$G_{eq} > G_1,\ G_{eq} > G_2 \ldots,\ G_{eq} > G_n$ |

**Example 3:** Find the equivalent resistance between the two points A & B.

**Solution:**

The resistances of 5Ω and 6 Ω are in series (as carry the same current).

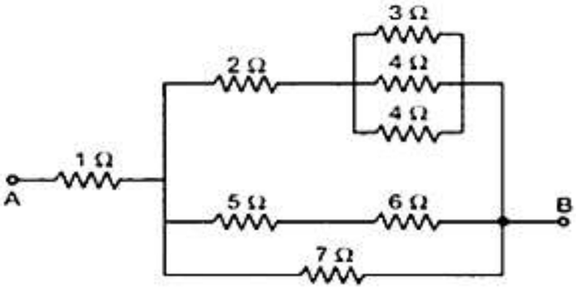



**Figure (2.6)(a)**

So equivalent resistance is 5+6 = 11 Ω

While resistances 3 Ω, 4 Ω, and 4 Ω are in parallel (as voltage across them same but current divides).

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = \frac{10}{12}$

∴ $R = \frac{12}{10} = 1.2\ \Omega$

Replacing these combinations and redraw the figure as shown in **Figure (2.6)(b)**.

Now again 1.2Ω and 2 Ω are in series so equivalent resistance is 2+1.2=3.2 Ω, while 11 Ω and 7 Ω are in parallel,

$\frac{1}{R} = \frac{1}{11} + \frac{1}{7} = \frac{18}{77}$ , ∴ $R = \frac{77}{18} = 4.277\ \Omega$

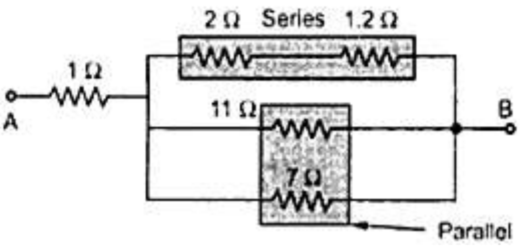



**Figure (2.6)(b)**

Replacing the respective combinations redraw the circuit as shown in **Figure (2.6)(c)**.

Now 3.2 Ω and 4.277Ω are in parallel.

$R = \frac{3.2 \times 4.277}{3.2 + 4.277} = 1.8304\Omega$

∴ $R_{AB} = 1 + 1.8304 = 2.8304\Omega$

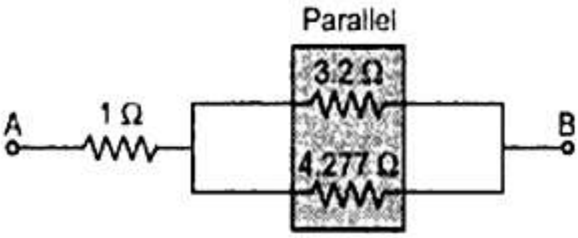



**Figure (2.6)(c)**

**Example 4:** Find the equivalent resistance between the two points A & B.

**Solution:**

Redraw the circuit as shown in **Figure (2.7)(b)**.

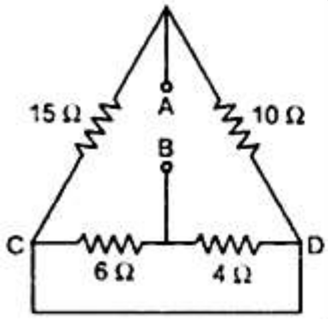



**Figure (2.7) (a)**

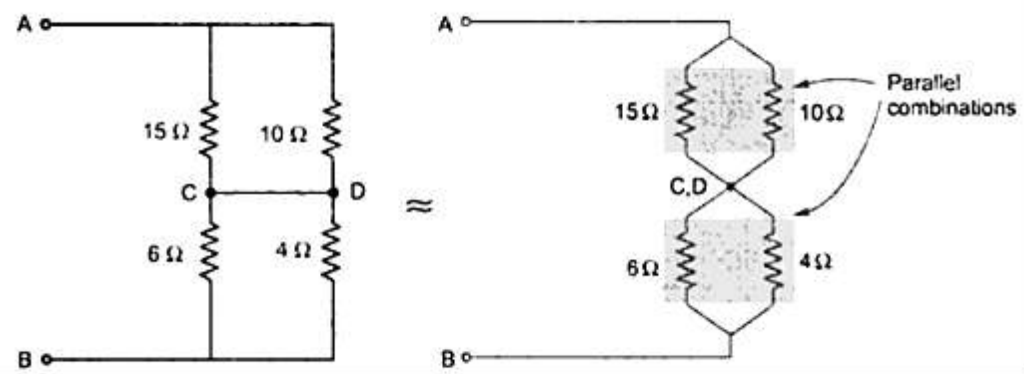



**Figure (2.7) (b)**

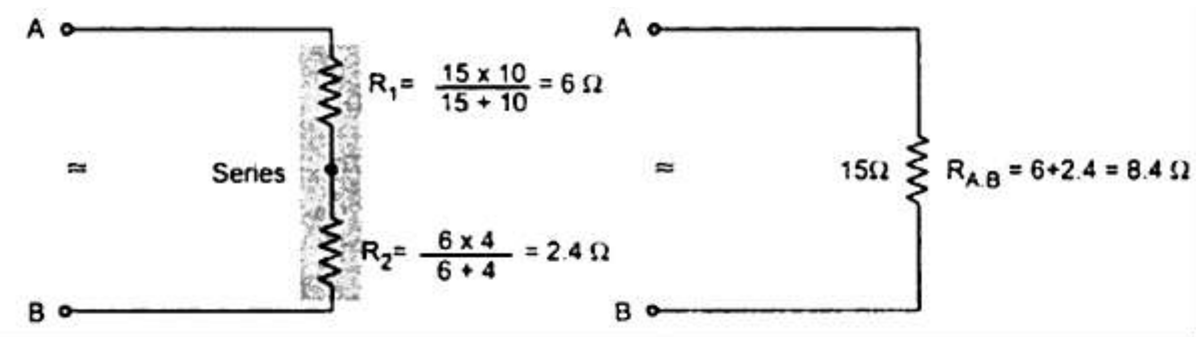



**Figure (2.7) (c)**

∴ $R_{AB}$ = **8.4 Ω**

# Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the network plays an important role.

## (i) Short Circuit

When any two points in a network are joined directly to each other with a thick metallic conducting wire, the two points are said to be short circuited. The resistance of such short circuit is zero.

The part of the network, which is short circuited is shown in the **Figure (2.8)**. The points A and B are short circuited. The resistance of the branch AB is $R_{SC} = 0\Omega$.
The current $I_{AB}$ is flowing through the short circuited path.

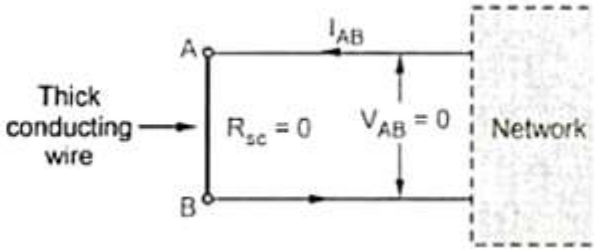



**Figure (2.8)**

According to Ohm's law, $V_{AB} = R_{SC} \times I_{AB} = 0 \times I_{AB} = 0\ V$

**Note:-** Thus, voltage across short circuit is always zero although current flows through the short circuited path.

## (ii) Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited. As there is no direct connection in an open circuit, **the resistance of the open circuit is** $\infty$.
The part of the network which is open circuited is shown in the **Figure (2.9)**. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{OC} = \infty\ \Omega$

There exists a voltage across the points AB called open circuit voltage, $V_{AB}$ but $R_{oc} = \infty\ \Omega$.

According to Ohm's law, $I_{OC} = \frac{V_{AB}}{R_{OC}} = \frac{V_{AB}}{\infty} = 0\ A$

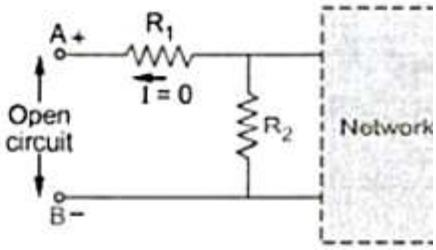



**Figure (2.9)**

**Note:-** Thus, current through open circuit is always zero through there exist a voltage across open circuit terminals.

## Redundant Branches and Combinations

The redundant means excessive and unwanted.

**Note:** If in a circuit there are branches or combinations of elements which do not carry any current then such branches and combinations are called redundant from circuit point of view.

The redundant branches and combinations can be removed and these branches do not affect the performance of the circuit.

The two important situations of redundancy which may exist in practical circuits are,

**Situation 1 :** Any branch or combination across which there exists a short circuit, becomes redundant as it does not carry any current.

To understand this, consider the combination of resistances and a short circuit as shown in the **Figure (2.10) (a) & (b)**.

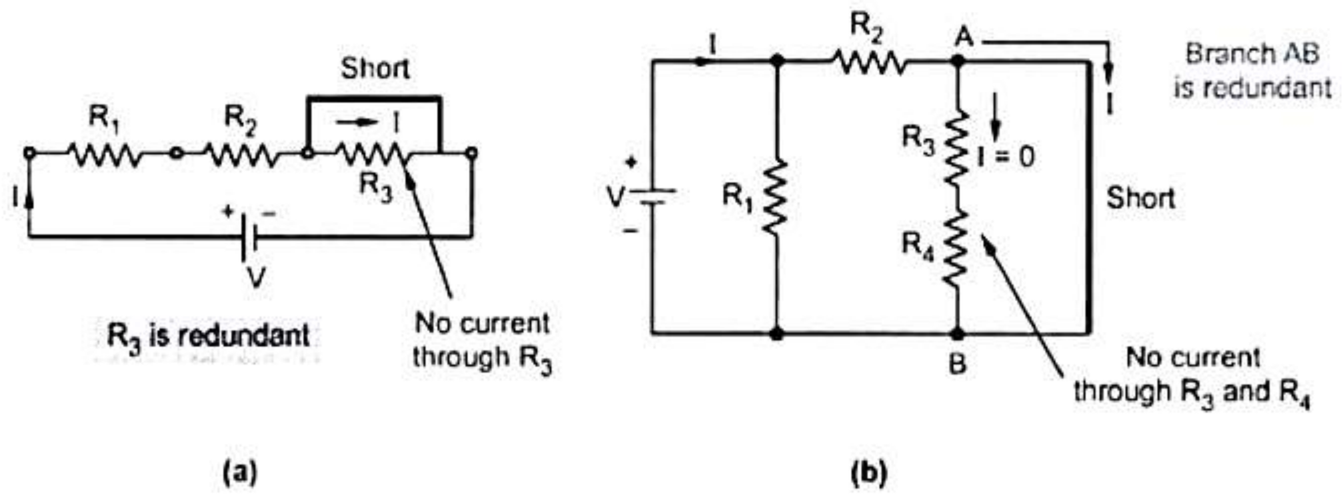



**Figure (2.10)**

In **Figure (2.10)** (a), there is short circuit across $R_3$. The current always prefers low resistance path hence entire current I passes through short circuit and hence resistance $R_3$ becomes redundant from the circuit point of view.

In **Figure (2.10)** (b), there is short circuit across combination of $R_3$ and $R_4$. The entire current flows through short circuit across $R_3$ and $R_4$ and no current can flow through combination of $R_3$ and $R_4$. Thus that combination becomes meaningless from the circuit point of view. Such combinations can be eliminated while analyzing the circuit.

**Situation 2 :** If there is open circuit in a branch or combination, it can not carry any current and becomes redundant.

In **Figure (2.11)** as there exists open circuit in branch BC, the branch BC and CD cannot carry any current and arc become redundant from circuit point of view.

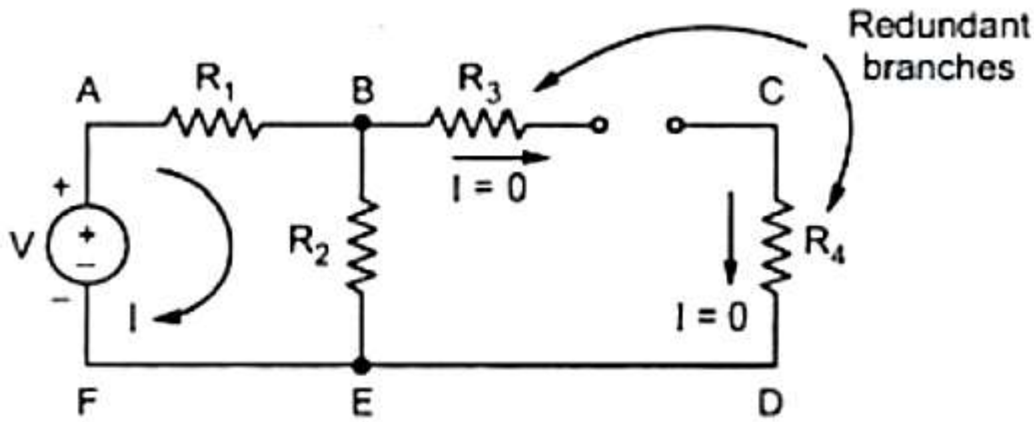



**Figure (2.11)**

## Voltage Division in Series Circuit Resistors

1- $I_T = I_1 = I_2$

2- $R_T = R_1 + R_2$

3- $V_T = V_1 + V_2$

V=IR (Ohm's law)

$\therefore V_T = I_T R_1 + I_T R_2 = I_T (R_1 + R_2)$

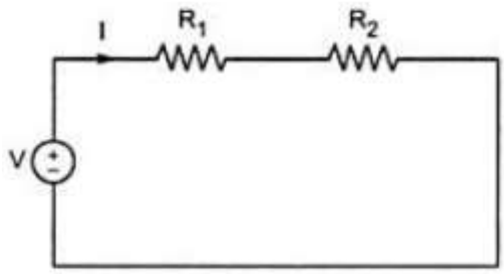



Figure (2.12)

$$\therefore \; I_T = \frac{V_T}{R_1 + R_2}$$

So

$$V_1 = I_T R_1 = \left(\frac{V_T}{R_1 + R_2}\right) R_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_T$$

Similarly

$$V_2 = I_T R_2 = \left(\frac{V_T}{R_1 + R_2}\right) R_2 = \left(\frac{R_2}{R_1 + R_2}\right) V_T$$

**Example 4:** Find the voltage across the three resistances shown in **Figure (2.13).**

**Solution:**

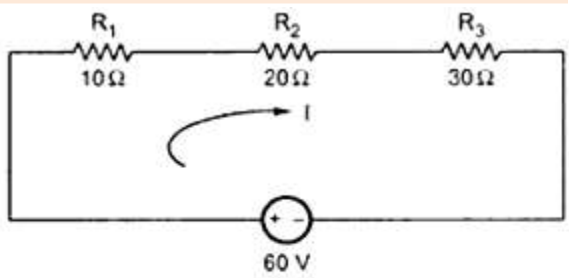



Figure (2.13)

$$I = \frac{V_T}{R_1 + R_2 + R_3} = \frac{60}{10 + 20 + 30} = 1A$$

$$V_{R1} = I.R_1 = \frac{V_T \times R_1}{R_1 + R_2 + R_3} = 1 \times 10 = 10V$$

$$V_{R2} = I.R_2 = \frac{V_T \times R_2}{R_1 + R_2 + R_3} = 1 \times 20 = 20V$$

$$V_{R3} = I.R_3 = \frac{V_T \times R_3}{R_1 + R_2 + R_3} = 1 \times 30 = 30V$$

## Current Division in Parallel Circuit Resistors

1- $V_T = V_1 = V_2$

2- $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

3- $I_T = I_1 + I_2$

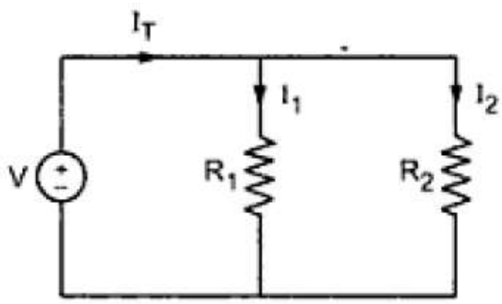


Figure (2.14)

$$i = \frac{V}{R} \text{ (Ohm's law)}$$

$$\therefore I_T = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V = \left(\frac{R_1 + R_2}{R_1 R_2}\right)V$$

$$\therefore V = \left(\frac{R_1 R_2}{R_1 + R_2}\right) I_T$$

$$I_1 = \frac{V}{R_1} = \left[\left(\frac{R_1 R_2}{R_1 + R_2}\right) I_T\right]\frac{1}{R_1} = \left(\frac{R_2}{R_1 + R_2}\right) I_T$$

$$I_2 = \frac{V}{R_2} = \left[\left(\frac{R_1 R_2}{R_1 + R_2}\right) I_T\right]\frac{1}{R_2} = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

**Example 5:** Find the $I_T$, $I_1$ & $I_2$. If $R_1 = 10\ \Omega$, $R_2 = 20\Omega$ & $V = 50$ V.

**Solution:**

The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{10 \times 20}{10 + 20} = 6.67\ \Omega$$

$$I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = 7.5 \text{ A}$$

The current distribution in parallel branches are,

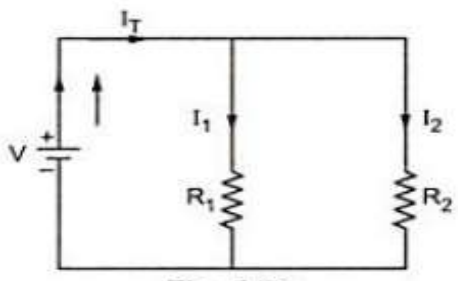


Figure (2.15)

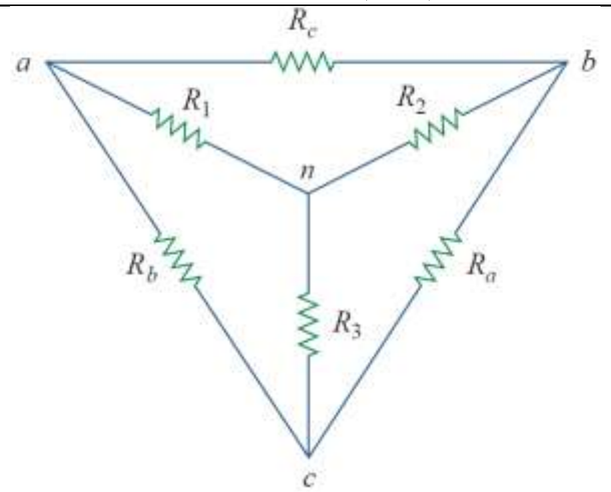
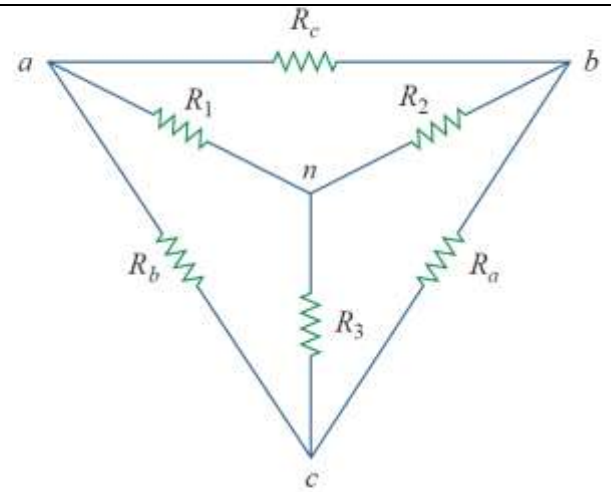
$$I_1 = I_T\left(\frac{R_2}{R_1 + R_2}\right) = 7.5 \times \left(\frac{20}{10+20}\right) = 5 \text{ A}$$

$$I_2 = I_T\left(\frac{R_1}{R_1 + R_2}\right) = 7.5 \times \left(\frac{10}{10+20}\right) = 2.5 \text{ A}$$ ( Note that, $I_T = I_1 + I_2$ or $7.5 = 5 + 2.5$)

## Star – Delta Conversion

If the resistors are neither series nor parallel then either star (some times called Wye (Y) or tee (T)) connected or delta (Δ) ( sometimes called pi(π))connected).

**Table (2.1)**

| Delta – Star (Δ-Y) | Star – Delta (Y-Δ) |
|---|---|
|   |   |
| $R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$ | $R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$ |
| $R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$ | $R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$ |
| $R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$ | $R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$ |

**Example 6:** convert the given delta in **Figure (2.16)** to star.

**Solution:**

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 5}{5+10+15} = \mathbf{1.67\ \Omega}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{15 \times 10}{5+10+15} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{5 \times 15}{5+10+15} = \mathbf{2.5\ \Omega}$$

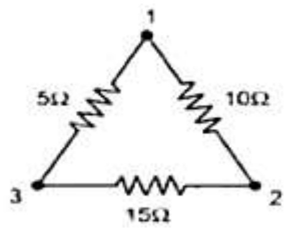



**Figure (2.16)**

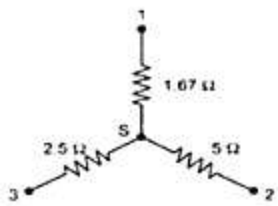



**Figure (2.17)**

**Example 7:** convert the given star in **Figure (2.18)** to delta.

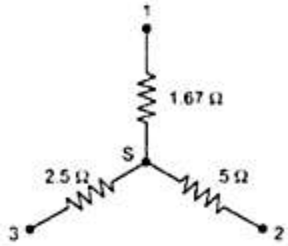



Figure (2.18)

**Solution:**

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{1.67 \times 5 + 5 \times 2.5 + 2.5 \times 1.67}{2.5} = \mathbf{10\ \Omega}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{1.67 \times 5 + 5 \times 2.5 + 2.5 \times 1.67}{1.67} = \mathbf{15\ \Omega}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{1.67 \times 5 + 5 \times 2.5 + 2.5 \times 1.67}{5} = \mathbf{5\ \Omega}$$

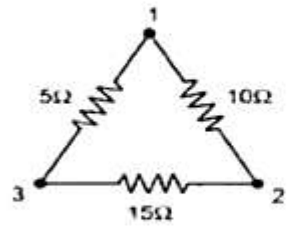



Figure (2.19)

**Example 8:** determine the resistance between the terminals X & Y for the circuit shown in **Figure (2.20).**

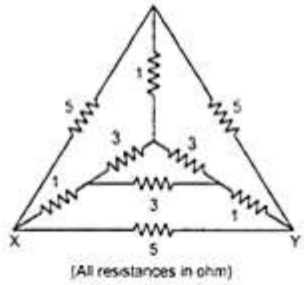



Figure (2.20)

**Solution:**

Converting inner delta to star,

$$\text{Each resistance} = \frac{3 \times 3}{3+3+3} = 1\ \Omega$$

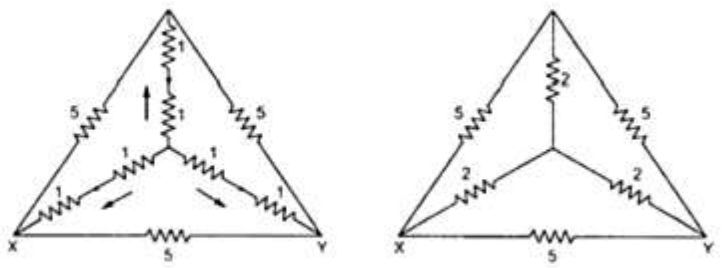



Figure (2.21)

Converting inner star to delta.

$$\text{Each resistance is } = \frac{2 \times 2 + 2 \times 2 + 2 \times 2}{2} = 6\ \Omega$$

All three parallel combinations,

$$5 \| 6 = \frac{5 \times 6}{5+6} = 2.7272\ \Omega$$

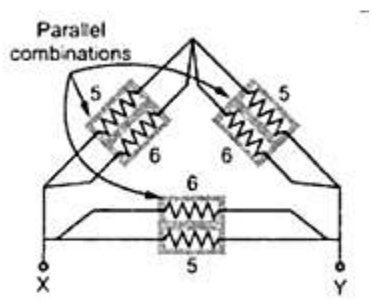



Figure (2.22)

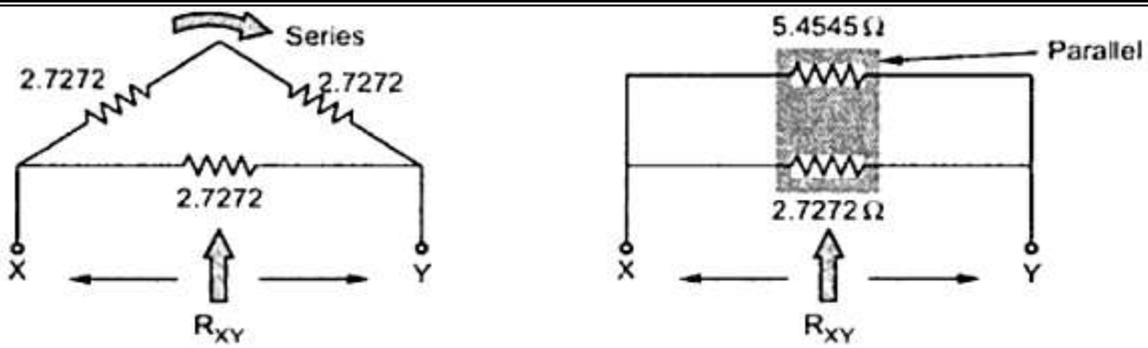



**Figure (2.23)**

$R_{XY}$ = 5.4545||2.7272= **1.8181 Ω**

**Example 9:** Calculate the equivalent resistance between the terminals A and B in the network shown in **Figure (2.24)(a).**

**Solution:**

The given circuit can be redrawn as shown in **Figure (2.24)(b).**

Convert the delta BCD is to its equivalent star, the circuit becomes as shown in **Figure (2.24)(c).**

$$R_{BN} = \frac{R_{BD}\,R_{BC}}{R_{BD} + R_{DC} + R_{CB}} = \frac{10 \times 10}{10 + 10 + 10} = \frac{10}{3}\ \Omega = R_{CN} = R_{DN}$$

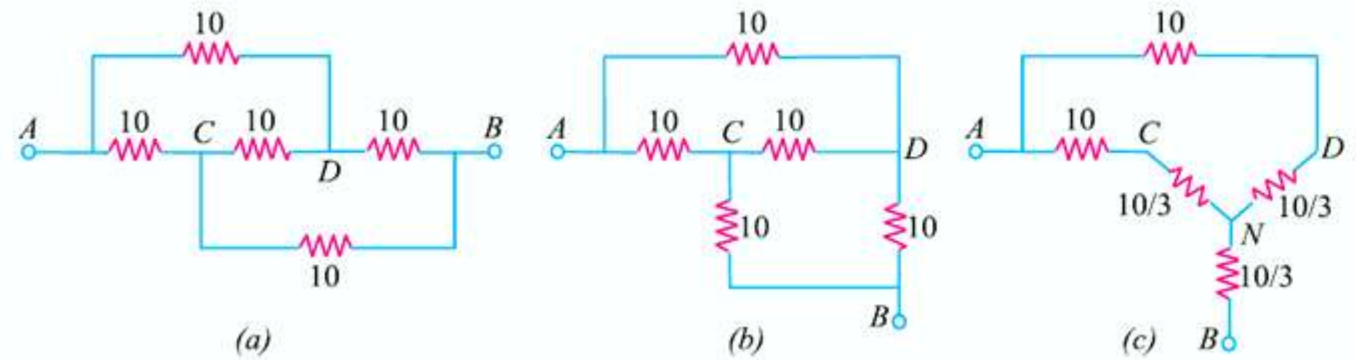



**Figure (2.24)**

As seen, there are two parallel paths between points A and N (as shown in **Figure (2.25)**), each having a resistance of (10 + 10/3) = 40/3 Ω.

Their combined resistance is, $\dfrac{\frac{40}{3} \times \frac{40}{3}}{\frac{40}{3} + \frac{40}{3}}$ = 20/3 Ω.

Hence, $R_{AB}$ = (20/3) + 10/3 = **10 Ω**.

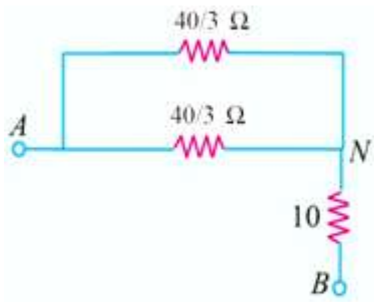



**Figure (2.25)**

**Example 10:** Calculate the current flowing through the 10 Ω resistor of **Figure (2.26)** by using star-delta transformation.

**Solution:**

The equivalent resistance between point A & B is ( 8 + 4 = 12Ω).

The equivalent resistance between point F & E is ( 17 + 13 = 30Ω).

The circuit is redrawn as shown in **Figure (2.27)**

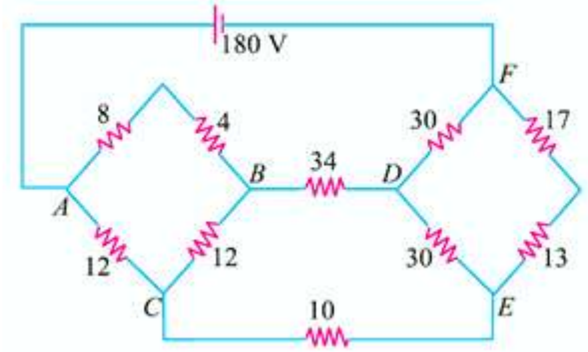



**Figure (2.26)**

It will be seen that there are two deltas in the circuit i.e. ABC and DEF. They have been converted into their equivalent stars as shown in **Figure (2.28)**.

Each arm of the delta ABC has a resistance of 12 Ω and each arm of the equivalent star has a resistance of 4 Ω.

Similarly, each arm of the delta DEF has a resistance of 30 Ω and the equivalent star has a resistance of 10 Ω per arm.

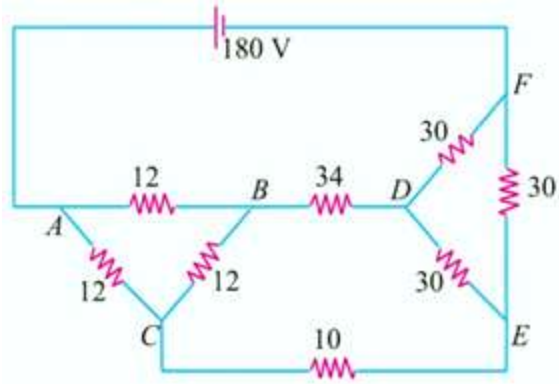



**Figure (2.27)**

The total circuit resistance between A and F = 4 + 48 ‖ 24 + 10 = 30 Ω.

Hence I = 180/30 = 6 A.

Current through 10 Ω resistor as given by current-divider rule = 6 × 48/(48 + 24) = 4 A.

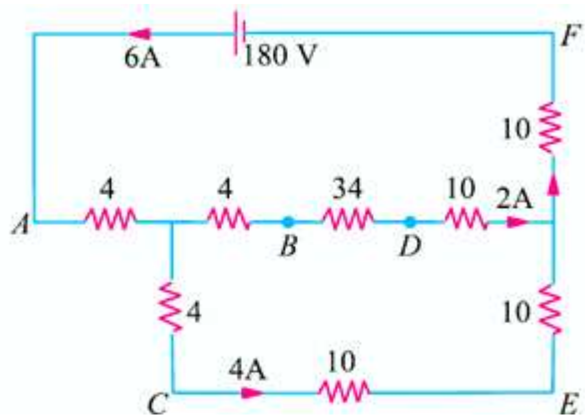



**Figure (2.28)**

**Note:-** for converting equal values resistors from delta to star or from star to delta, then the equivalent resistors are equals.

# Home Works

**H.W.(1):-** Find the current in the 17 Ω resistor in the network shown in **Figure (2.29)** by using (a) star/delta conversion. **[10/3A]**

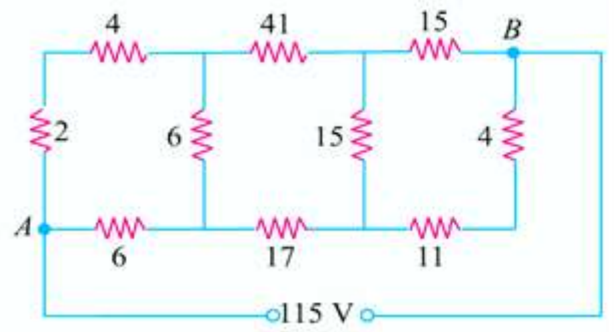



**Figure (2.29)**

**H.W.(2):-** Determine the resistance between points A and B in the network of **Figure (2.30)**. **[4.23 Ω]**

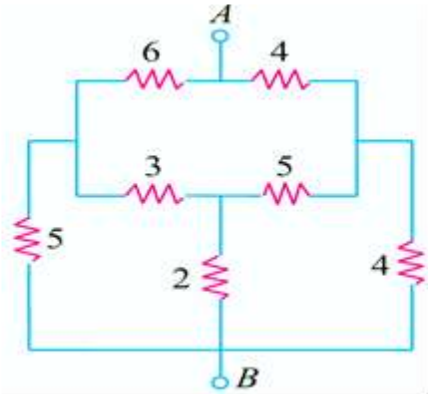



**Figure (2.30)**

**H.W.(3):-** Find the current and power supplied by the 40V source in the circuit shown in **Figure (2.31)**. **[0.5A, 20W]**

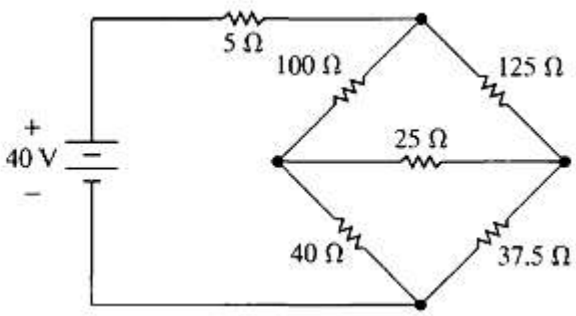



**Figure (2.31)**

**H.W.(4):-** Find the equivalent resistance between the terminals a & b in the circuits shown in **Figure (2.32)(a) to (k)**.

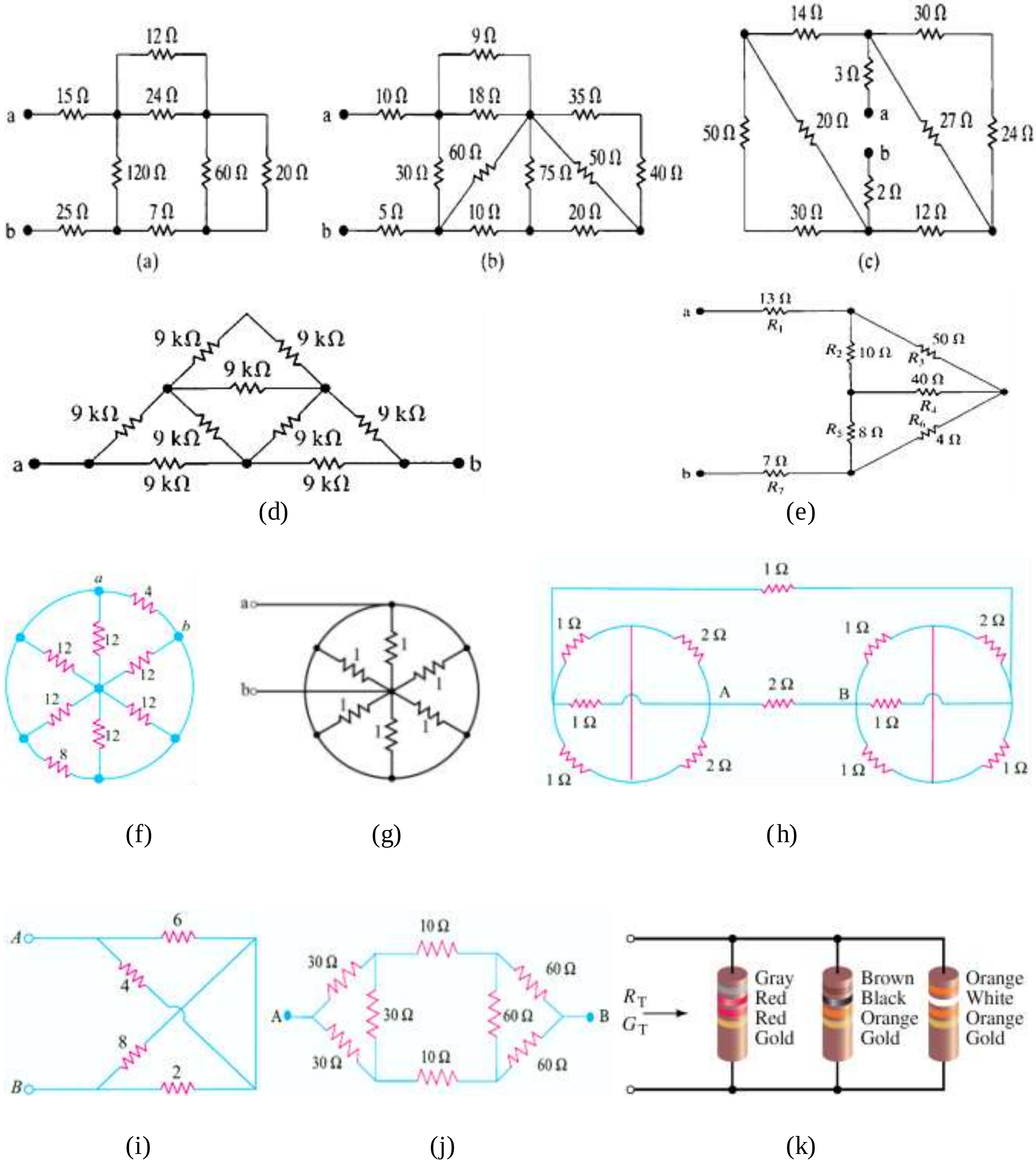



**Figure (2.32)**

**[Answer: (a)64Ω, (b)30Ω, (c)20Ω, (d)10 KΩ, (e)33Ω, (f)2Ω, (g)0.168Ω, (h)1.034Ω, (i) 4Ω, (j)50Ω]**

**H.W.(5):-** Find the equivalent resistance as seen from voltage or current sources in the circuits shown in **Figure (2.33) from (a) to (j)**.

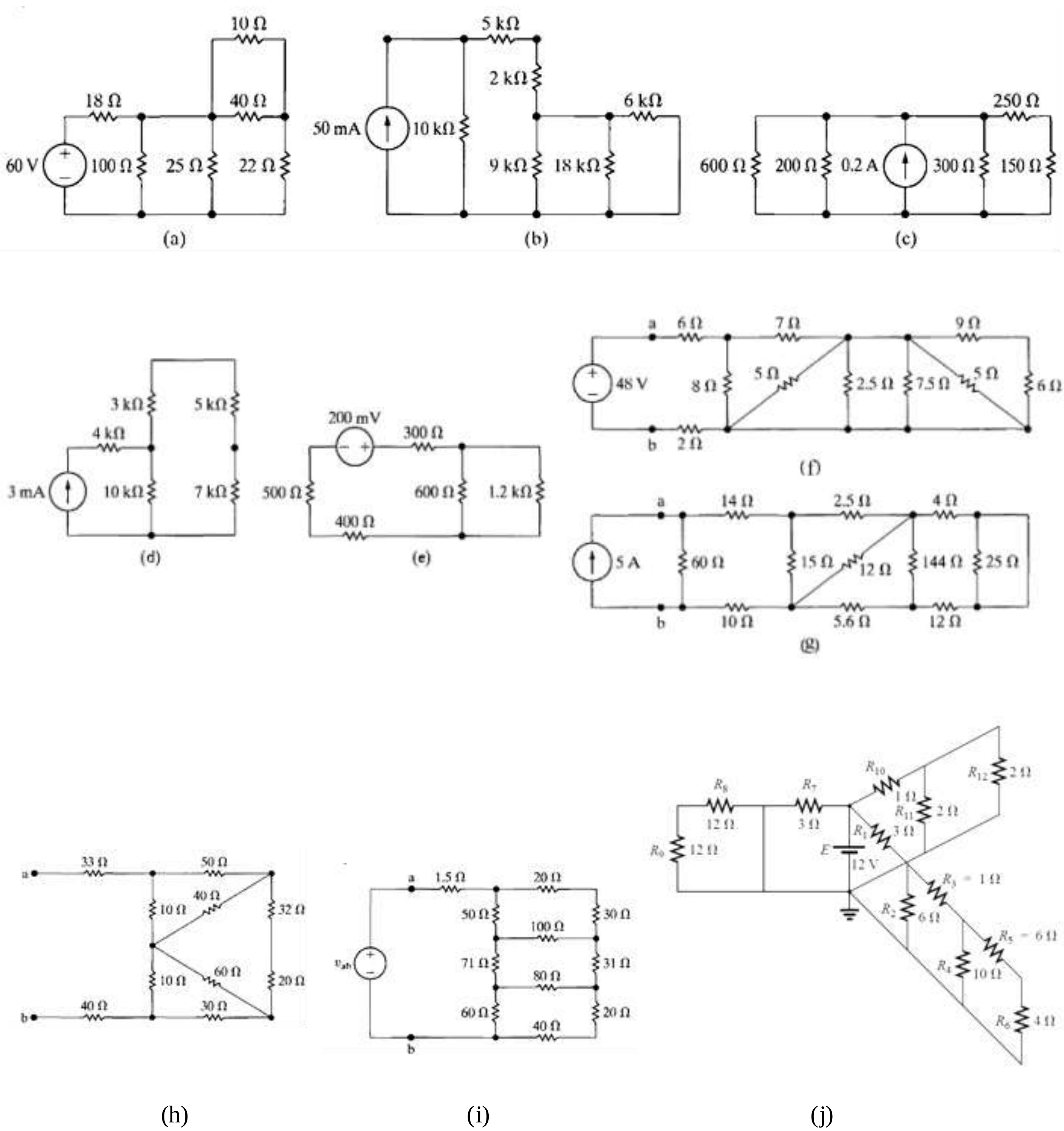



**Figure (2.33)**

**H.W.(6):-** Find the equivalent resistance between terminals (a) a & b (b) a &c (c) b& c, of circuits shown in **Figure (2.34)**.

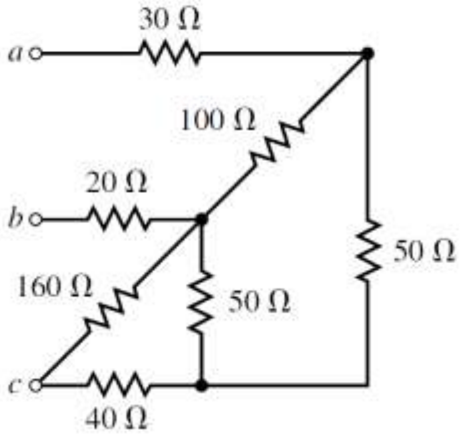



**Figure (2.34)**

**[Answer :Rab = 97.4Ω, Rac = 95.88Ω, Rbc = 72.2Ω]**

**H.W.(7):-** For the circuit in **Figure (2.35 ),** if $i_x$ = 5A then find $v_1$, $v_2$, $i_s$ & $i_y$.

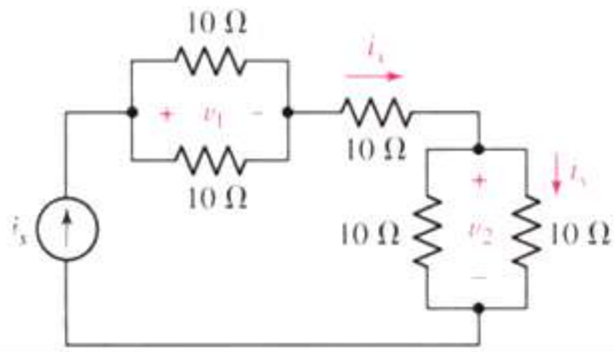



**Figure (2.35 )**

**H.W.(8):-** Find $i_o$ the circuit in **Figure (2.36)**.

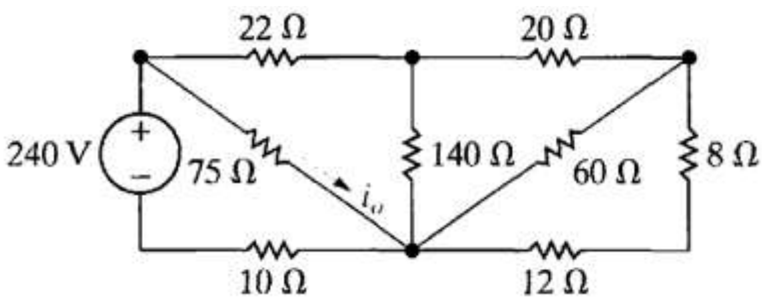



**Figure (2.36)**

**[Answer :$I_o$ = 2.4 A]**

**H.W.(9):-** The current in the 12Ω resistor of the circuit in **Figure (2.37)** is 1A. Find the value of $V_g$.

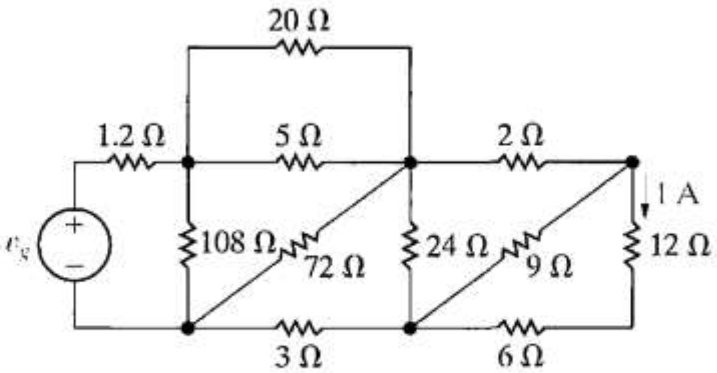



**Figure (2.37)**

**H.W.(10):-** Find I the circuit in **Figure (2.38)**.

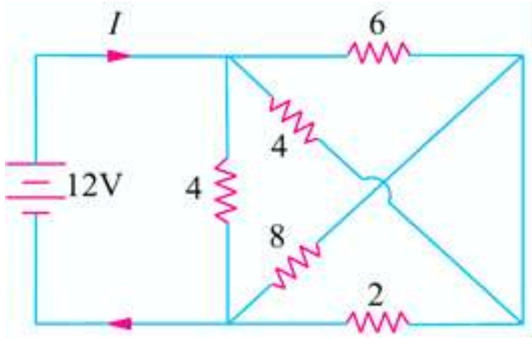



**Figure (2.38)**

**[Answer : I = 6A]**

**H.W.(11):-** Find I & $I_1$ the circuit in **Figure (2.39)**.

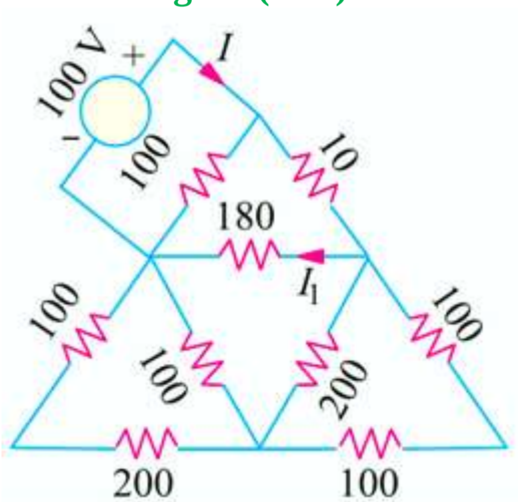



**Figure (2.39)**

**[Answer : I = 2.013A, I1 =−499.3mA]**

**H.W.(12):-** In the circuit shown in **Figure (2.40)**, calculate (a) current I (b) current $I_1$ and (c) $V_{AB}$. All resistances are in ohms. **[Answer : (a) 4 A (b) 0.25 A (c) 4 V]**

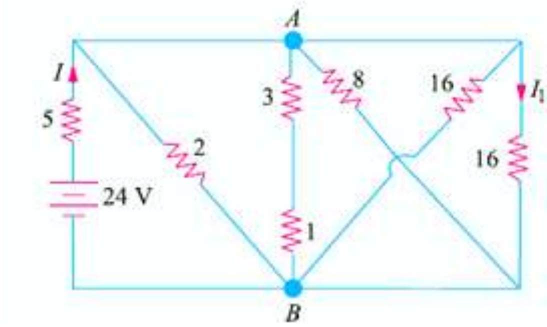



**Figure (2.40)**

**H.W.(13):-** Determine the unknown resistors of **Figure (2.41)** given the fact that $R_2 = 5R_1$ and $R_3 = (1/2)R_1$.

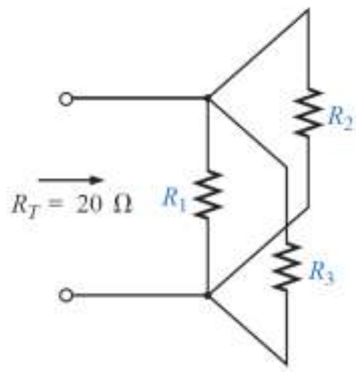



**Figure (2.41)**

**H.W.(14):-** Determine $R_1$ for the network of **Figure (2.42)**.

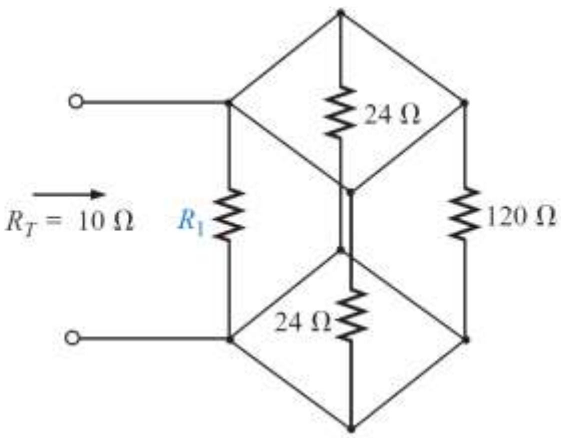
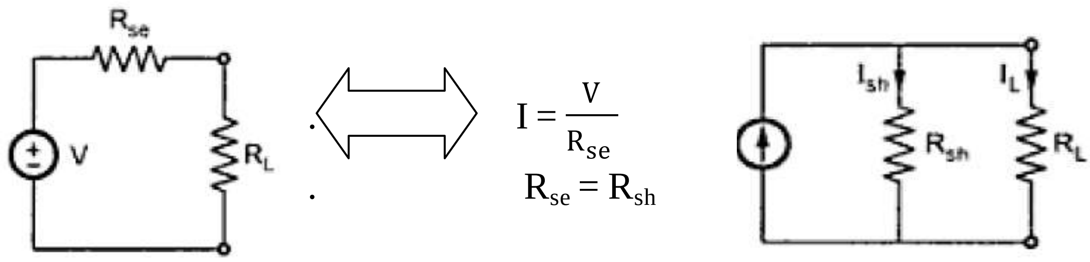



**Figure (2.42)**

## Source Transformation

Consider a practical voltage source having internal resistance ($R_{se}$) as shown in **Figure (2.43)(a)**, connected to load ($R_L$).

Now we can replace voltage source by equivalent current source as shown in **Figure (2.43)(b)**.




$$I = \frac{V}{R_{se}}$$

$$R_{se} = R_{sh}$$

**(a)** **(b)**

**Figure (2.43)**

> **Note:** The two sources are said to be ***equivalent***, if they supply equal load current to the load, with same load resistance connected across it's terminals.

For **(2.43)(a)**, $I_L = \dfrac{V}{R_{se} + R_L}$ ,

For **(2.43)(b)**, $I_L = I \times \dfrac{R_{sh}}{R_{sh} + R_L}$ , $R_{se} = R_{sh}$

And $I_L$ must be equal for both cases.

Directions of transformed sources, shown in **Figure (2.44)(a), (b), (c) and (d).**

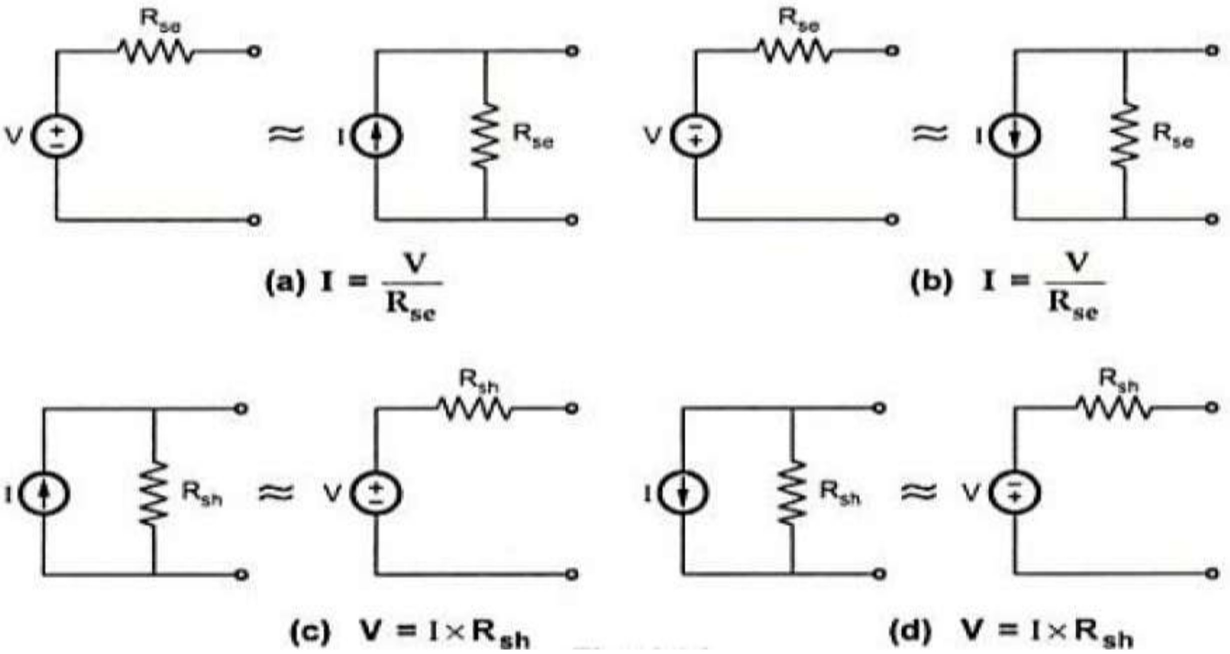



**Figure (2.44)**

- Practical voltage source has internal resistance ($R_{se}$) connected in series with source.
- Practical current source has internal resistance ($R_{sh}$) connected in parallel with source.
- Voltage source can be converted into equivalent current source (using Ohm's law) and vice versa.
- When convert source, you must take into account the polarity of sources.
  When converting voltage source to current source, the current source arrow is directed from –ve terminal to +ve terminal of voltage source. While converting current source to voltage source, polarities of voltage source is always +ve terminal at top of current source arrow and –ve terminal at bottom of current source arrow.
- Source transformation also applies to dependent sources.
- An ideal voltage source with R = 0 cannot be replaced by a finite current source.
- An ideal current source with R = ∞ cannot be replaced by a finite voltage source.

**Example 9:** transform the ideal voltage sources shown in **Figure(2.45)** to ideal current sources.

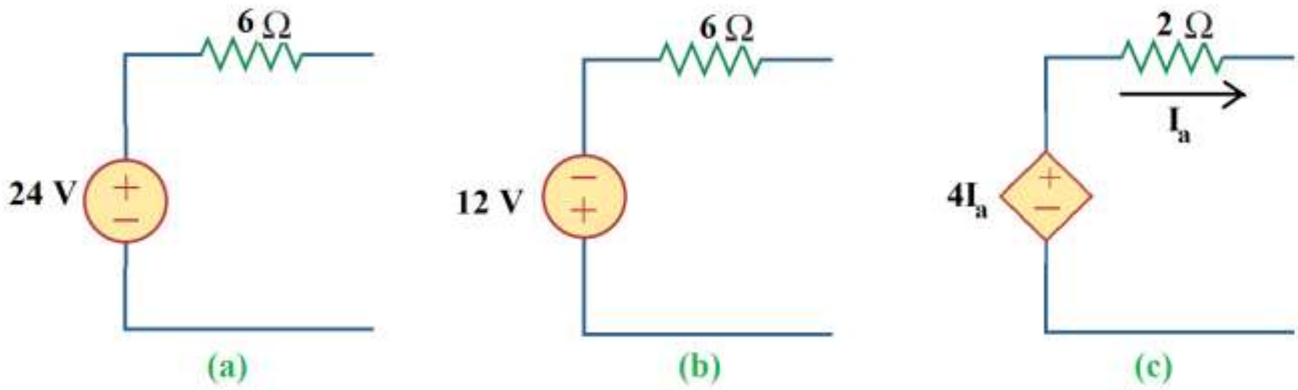



**Figure (2.45)**

**Solution:**

$$(a)\ I = \frac{V}{R_{se}} = \frac{24}{6} = 4\ A, \qquad R_{sh} = R_{se} = 6\ \Omega$$

$$(b)\ I = \frac{V}{R_{se}} = \frac{12}{6} = 2\ A, \qquad R_{sh} = R_{se} = 6\ \Omega$$

$$(c)\ I = \frac{V}{R_{se}} = \frac{4I_a}{2} = 2I_a\ A, \qquad R_{sh} = R_{se} = 2\ \Omega$$

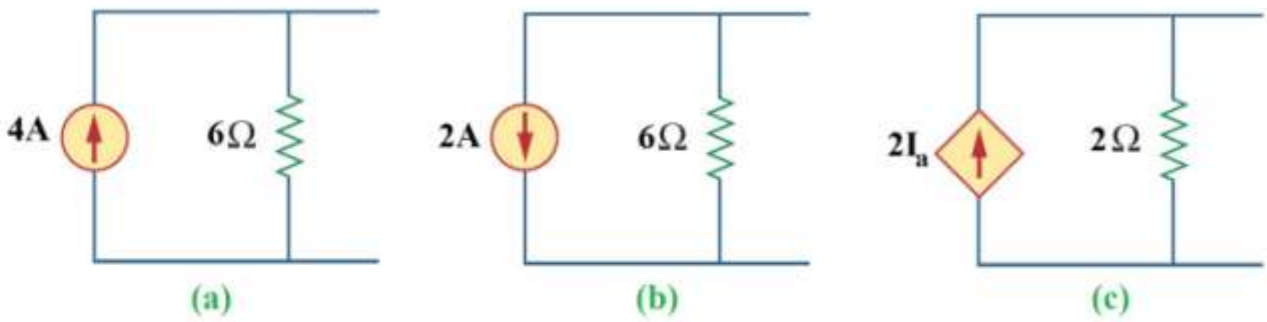



**Figure (2.46)**

**Example 10:** transform the ideal current sources shown in **Figure(2.7)** to ideal voltage sources.

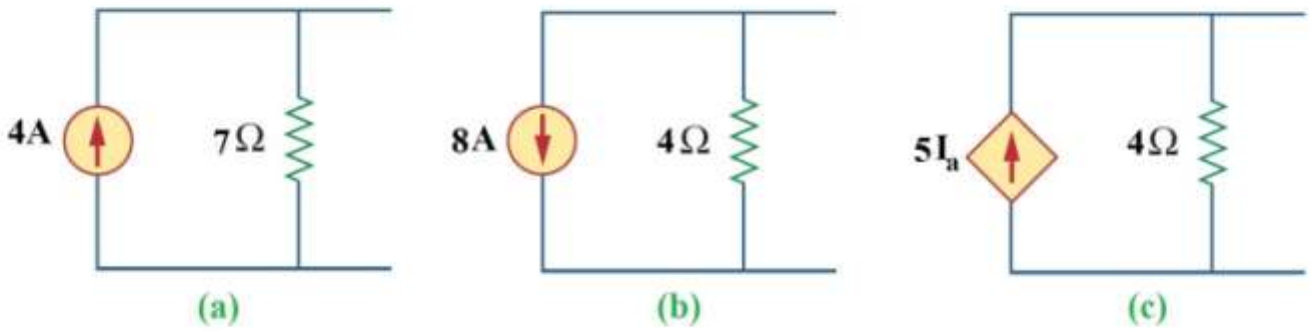



**Figure (2.47)**

**Solution:**

(a) $V = I.R_{sh} = 4 \times 7 = 28V$, $R_{se} = R_{sh} = 7\ \Omega$

(b) $V = I.R_{sh} = 8 \times 4 = 32V$, $R_{se} = R_{sh} = 4\ \Omega$

(c) $V = I.R_{sh} = 5I_a \times 4 = 20I_aV$, $R_{se} = R_{sh} = 4\ \Omega$

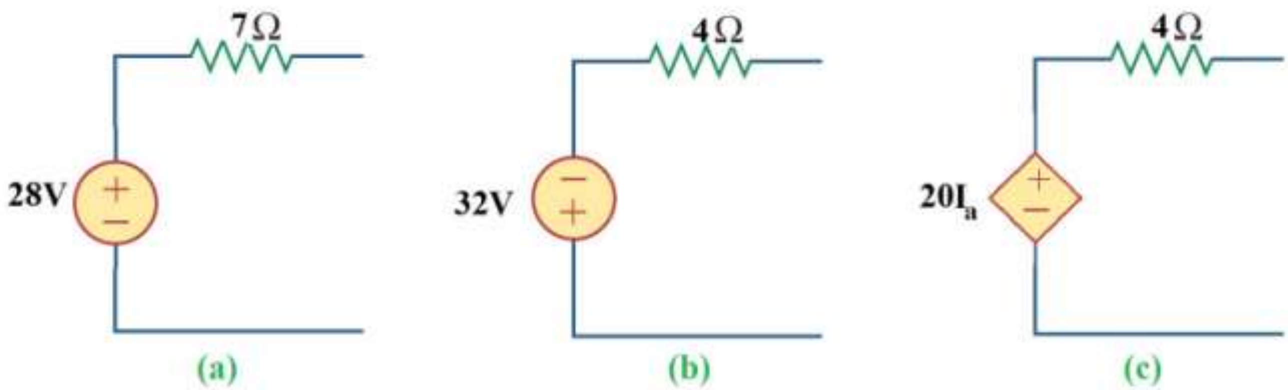



**Figure (2.48)**

**Example 11:** transform the network shown in **Figure(2.49)** to single ideal voltage sources.

**Solution:**

- The two current sources are in parallel. Hence they have an equivalent,
  $I = I_1 + I_2 = 9 - 3 = 6$ A
  With two resistors in parallel. Hence the equivalent resistor is,
  $$R = \frac{3 \times 6}{3+6} = 2\Omega$$

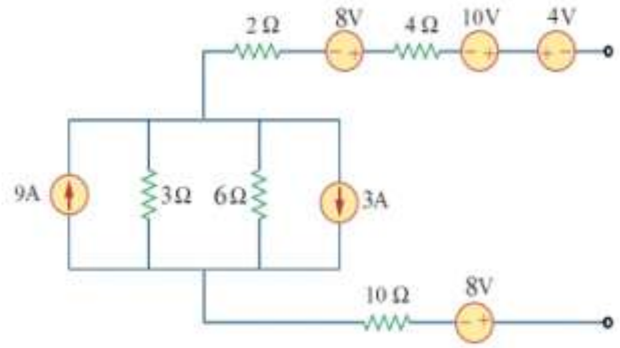



**Figure (2.49)**

- The three voltage sources are in series. Hence they have an equivalent,
  $V = V_1 + V_2 + V_3 = 8 + 10 - 4 = 14V$
  With two resistors in series. Hence the equivalent resistor is,
  $R = 2 + 4 = 6\Omega$
  So the equivalent network becomes as shown in **Figure(2.50).**

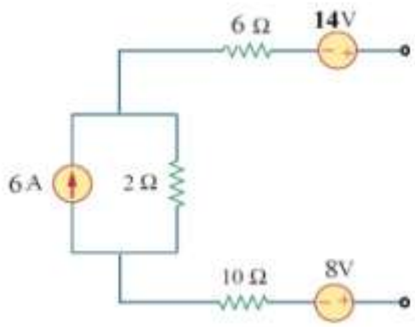



**Figure (2.50)**

- Convert the (6A) current source with (2Ω) resistor in parallel to a voltage source with (2Ω) resistor in series as shown in **Figure(2.51).**
  $V = IR = 6 \times 2 = 12V$

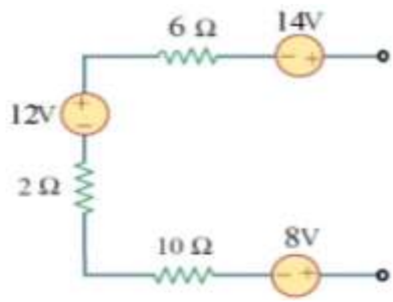



**Figure (2.51)**

- **Figure(2.50)** shows three voltage sources in series and three resistors in series also. Hence they have an equivalent,
  $V = V_1 + V_2 + V_3 = 14 + 12 - 8 =$ **18V**
  $R = R_1 + R_2 + R_3 = 6 + 2 + 10 =$ **18Ω**
  The equivalent network is shown in **Figure(2.52)**.

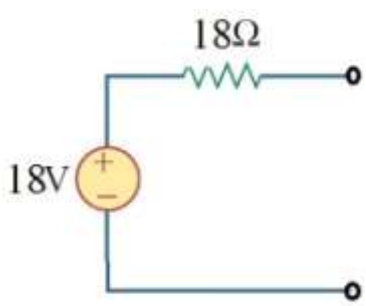
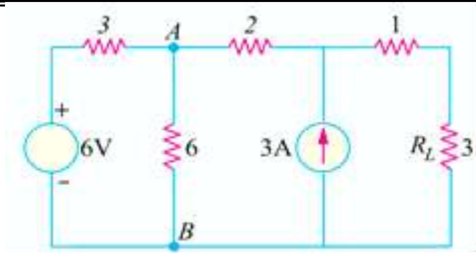



**Figure (2.52)**

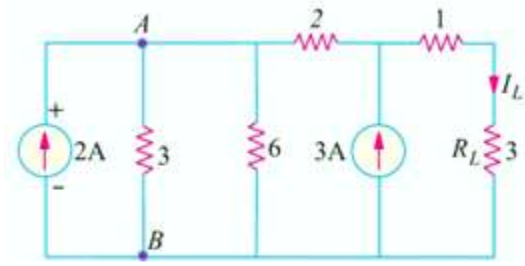
**Example 12:** Use Source Conversion technique to find the load current I in the circuit of **Figure(2.53)**.



**Figure(2.53)**

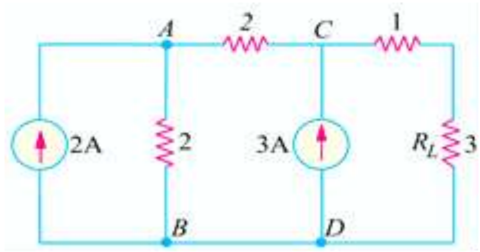
**Solution:**

The 6V voltage source with a series resistance of 3 Ω has been converted into an equivalent 2 A current source with 3 Ω resistance in parallel as shown in **Figure(2.54)**.



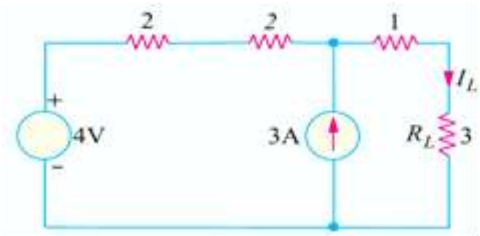
**Figure(2.54)**

In **Figure(2.54)** the two parallel resistances of 3 Ω and 6 Ω can be combined into a single resistance of 2 Ω as shown in **Figure(2.55)**.



**Figure(2.55)**

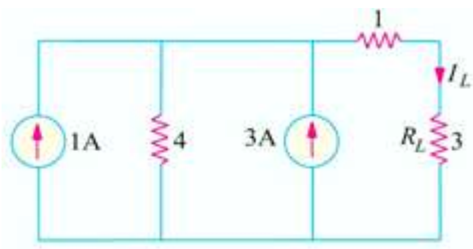
In **Figure(2.55)** the two current sources cannot be combined together because of the 2 Ω resistance present between points A and C. To remove this hurdle, we convert the 2 A current source in parallel with 2Ω resistor into the equivalent 4 V voltage source in series with 2Ω resistor as shown in **Figure(2.56)**.



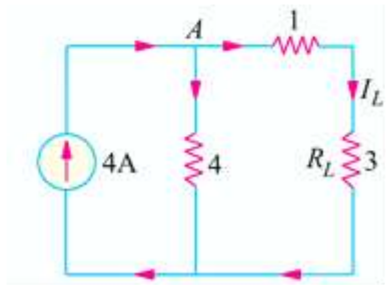
**Figure(2.56)**

In **Figure(2.56)** the 4 V voltage source with a series resistance of (2 + 2) = 4 Ω can again be converted into the equivalent current source as shown in **Figure(2.57)**.

In **Figure(2.57)** the two current sources can be combined into a single 4-A source as shown in **Figure(2.58)**. The 4-A current is divided into two equal parts at point A because each of the two parallel paths has a resistance of 4 Ω. Hence $I_1$ = **2 A**.



**Figure(2.57)**



**Figure(2.58)**

Lecture (3)

# Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist **Gustav Robert Kirchhoff** (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

## 1- Kirchhoff's current law (KCL)

**Kirchhoff's current law (KCL)** states that the algebraic sum of currents entering a node is zero.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

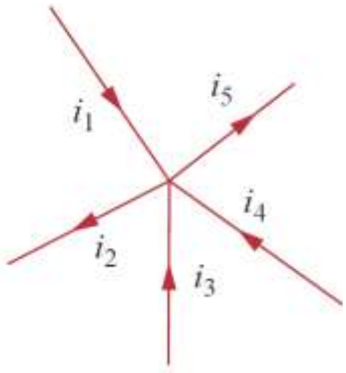



Figure (3.1)

Where N is the number of branches connected to the node and $i_n$ is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

So for Figure (3.1):

$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$

By rearranging the terms, we get

$i_1 + i_3 + i_4 = i_2 + i_5$

So Kirchhoff's current law also state that:

**Kirchhoff's current law (KCL)** states that the sum of the currents entering a node is equal to the sum of the currents leaving the node.

## 2- Kirchhoff's voltage law (KVL)

**Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0$$

Where M is the number of voltages in the loop (or the number of branches in the loop) and $v_m$ is the *m*th voltage.

So for **Figure (3.2)**:

$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$

Rearranging terms gives

$v_2 + v_3 + v_5 = v_1 + v_4$

which may be interpreted as:

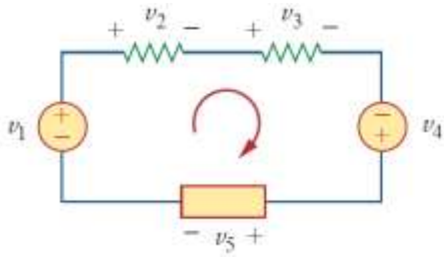



**Figure (3.2)**

**Kirchhoff's voltage law (KVL)** states that the Sum of voltage drops = Sum of voltage rises

## Determination of signs for sources and elements for KVL

1- **For voltage source**, if we start the loop from negative terminal to positive terminal, then the voltage should be given negative sign. On other hand, If we start the loop from positive terminal to negative terminal, then the voltage should be given positive sign.

2- **For elements (resistances),** if we start the loop through the resistor in the same direction as the current through it, then the voltage should be given positive sign. **On other hand,** if we start the loop through the resistor in the opposite direction as the current through it, then the voltage should be given negative sign.

**Note:** you can also reverse the two above assumption for solving problems, as stated below, 1- **For voltage source**, if we start the loop from negative terminal to positive terminal, then the voltage should be given positive sign. On other hand, If we start the loop from positive terminal to negative terminal, then the voltage should be given negative sign.

2- **For elements (resistances),** if we start the loop through the resistor in the same direction as the current through it, then the voltage should be given negative sign. **On other hand,** if we start the loop through the resistor in the opposite direction as the current through it, then the voltage should be given positive sign.

**Example 1:** Find the voltage between terminals a & b in **Figure (3.3)**.

**Solution:**

$-V_{ab} + V_1 + V_2 - V_3 = 0$

$\therefore V_{ab} = V_1 + V_2 - V_3$

If $V_1 = 50$ V, $V_2 = 30$ V & $V_3 = 45$ V

Then $V_{ab} = 50 + 30 - 45 = 35$ V

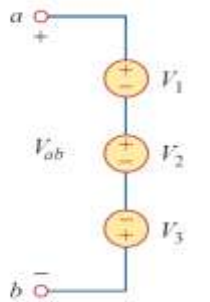



**Figure (3.3)**

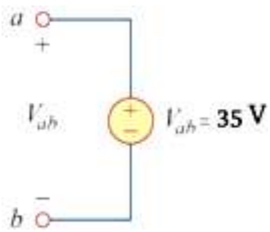



**Figure (3.4)**

So we can replace the three series sources by a single source equal to (35 V).

**Example 2:** for the circuit in **Figure (3.5)(a),** find voltages $v_1$ & $v_2$.

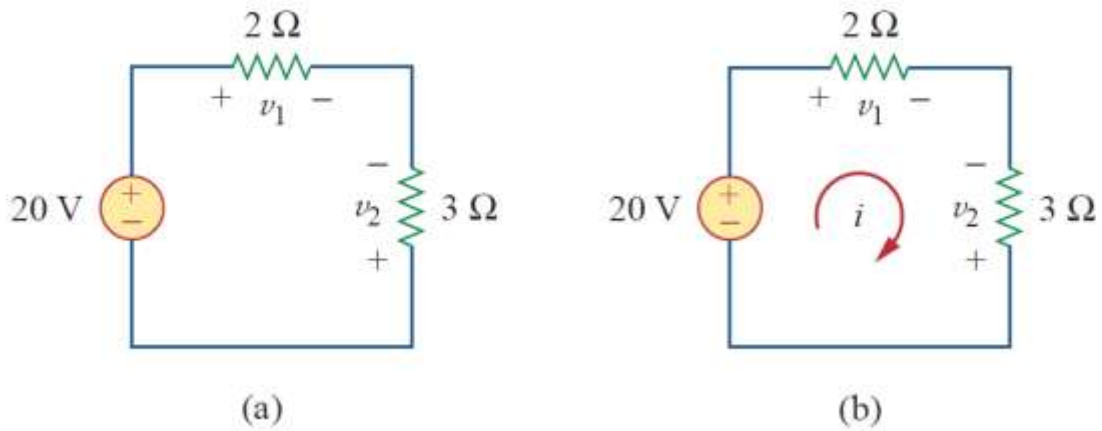



**Figure (3.5)**

**Solution:**

To find and we apply Ohm's law and Kirchhoff's voltage law.
Assume that current $i$ flows through the loop as shown in **Figure (3.5)(b).**
From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \qquad \ldots(1)$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \qquad \ldots(2)$$

Substituting Eq. (1) into Eq. (2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \Rightarrow i = 4\text{ A}$$

Substituting $i$ in Eq. (1) finally gives,

$$v_1 = 8\text{ V}, \qquad v_2 = -12\text{ V}$$

**Example 3:** Determine $v_o$ and $i$ in the circuit shown in **Figure (3.6)(a).**

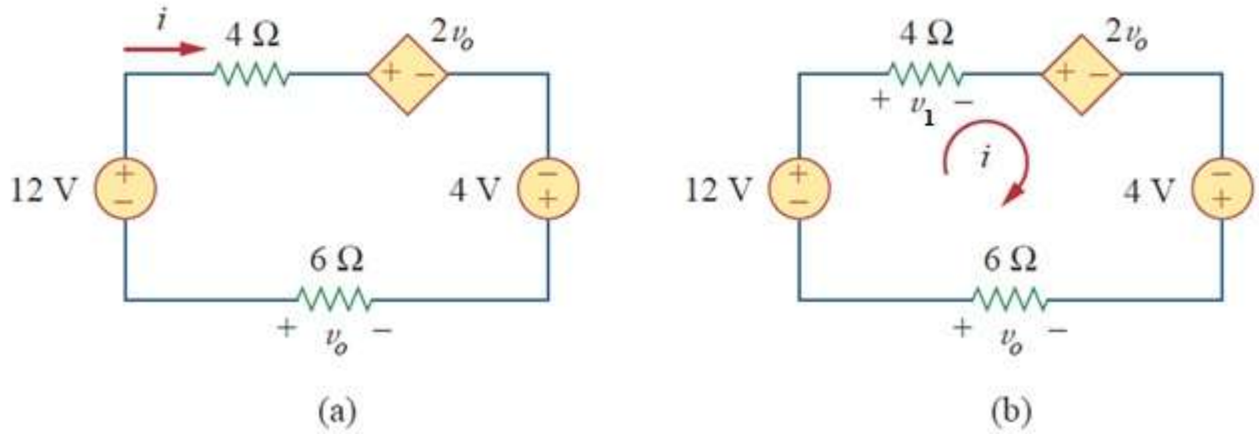



**Figure (3.6)**

**Solution:**

We apply KVL around the loop as shown in **Figure (3.6)(b)**. The result is

$-12 + v_1 + 2v_o - 4 - v_o = 0$, where $v_1 = 4i$, $v_o = -6i$

$$\therefore \qquad -12 + 4i + 2v_o - 4 + 6i = 0 \qquad \ldots(1)$$

Applying Ohm's law to the 6- resistor gives

$$v_o = -6i \qquad \ldots(2)$$

Substituting Eq. (2) into Eq. (1) yields

$$-16 + 10i - 12i = 0 \Rightarrow i = -8\text{ A}$$

and $\qquad v_o = 48$ V.

**Example 4:** What is the voltage $V_s$ across the open switch in the circuit shown in **Figure (3.7),** use Kirchhoff's laws.

**Solution:**

Apply KVL starting from point A (counter clock wise direction),

$+V_s + 10 - 20 - 50 + 30 = 0$

$\therefore\ V_s =$ **30 V**

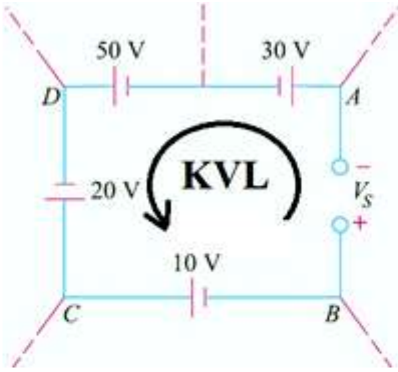



**Figure (3.7)**

**Example 5:** Find the unknown voltage $V_1$ in the circuit of **Figure (3.8),** use Kirchhoff's laws.

**Solution:**

Apply KVL through loop ABCDEFA,

$-v_1 + 40 - v_2 - v_3 = 0$

$v_2 = 2i_2 = 2 \times 4 = 8$ V

$v_3 = 3i_3 = 3 \times 16 = 48$ V

$\therefore -v_1 + 40 - 8 - 48 = 0$

$\therefore v_1 =$ **– 16 V**

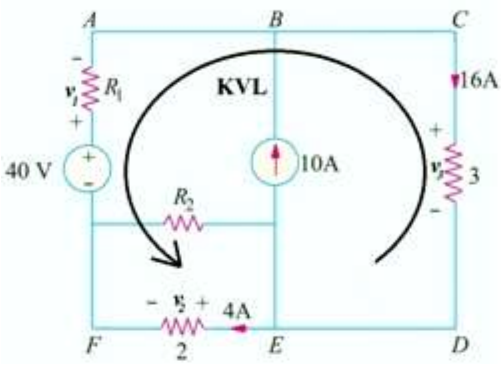



**Figure (3.8)**

**Example 6:** Using Kirchhoff's Current Law and Ohm's Law, find the magnitude and polarity of voltage V in **Figure (3.9).**

**Solution:**

- Let us arbitrarily choose the directions of $I_1$, $I_2$ and $I_3$ and polarity of V as shown in **Figure (3.10).**
- Applying KCL to node A, we have
  $-I_1 + 30 + I_2 - I_3 - 8 = 0$ Or $I_1 - I_2 + I_3 = 22$

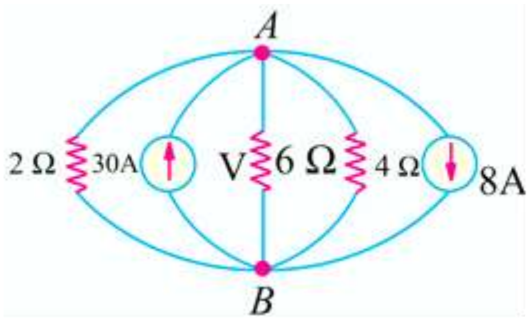



**Figure (3.9)**

Applying Ohm's law to the three resistive branches in **Figure (3.10)**, we have

$I_1 = \frac{V}{2}$ , $I_2 = -\frac{V}{6}$ and $I_3 = \frac{V}{4}$

Substituting these values in (i) above, we get

$\frac{V}{2} + \frac{V}{6} + \frac{V}{4} = 22 \quad \Longrightarrow \quad \therefore V =$ **24 V**

$\therefore\ I_1 = V/2 = 24/2 = 12$ A, $I_2 = -24/6 = -4$ A, $I_3 = 24/4 = 6$ A

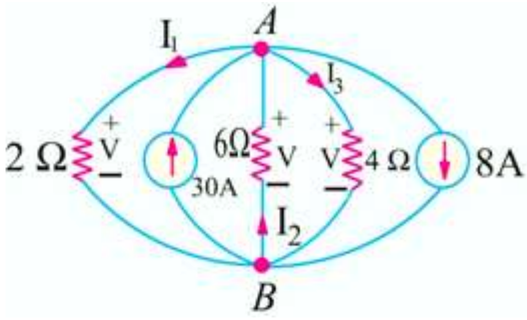



**Figure (3.10)**

**Example 7:** For the circuit in **Figure (3.11)** find $V_{CE}$ & $V_{AG}$ using Kirchhoff's laws.

**Solution:**

Redraw the circuit as shown in **Figure (3.12)**.

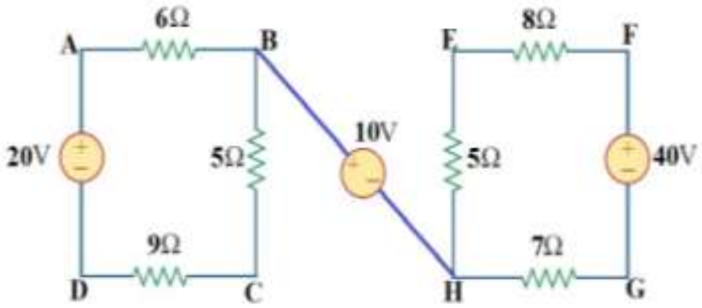



**Figure (3.11)**

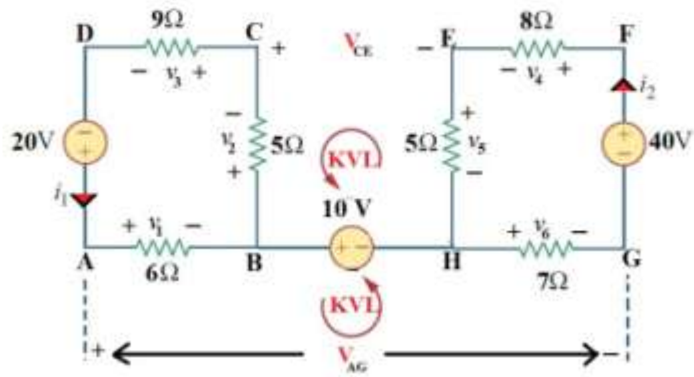



**Figure (3.12)**

❖ To find VCB & VBH, we must find $i_1$ & $i_2$.

$$i_1 = \frac{20}{6+5+9} = \mathbf{1\ A} \qquad \& \qquad i_2 = \frac{40}{8+5+7} = \mathbf{2\ A}$$

❖ To find $V_{CE}$, apply KVL for loop ECBHE as shown in **Figure (3.12).**

$-V_{CE} - v_2 + 10 - v_5 = 0$

$V_{CE} = -v_2 + 10 - v_5$

$v_2 = 5\,i_1 = 5 \times 1 = 5V$ & $v_5 = 5\,i_2 = 5 \times 2 = 10V$

$\therefore V_{CE} = -5 + 10 - 10 = \mathbf{-5\ V}$

❖ To find $V_{AG}$, apply KVL on loop ABHGA shown in **Figure (3.12).**

$-V_{AG} + v_1 + 10 + v_6 = 0$

$V_{AG} = v_1 + 10 + v_6$

$v_1 = 6\,i_1 = 6 \times 1 = 6V$ & $v_6 = 7\,i_2 = 7 \times 2 = 14V$

$\therefore V_{AG} = 6 + 10 + 14 = \mathbf{30\ V}$

**Note:**

❖ $V_{CE} = V_{CB} + V_{BH} + V_{HE}$ Or $V_{CE} = V_{CB} + V_{BH} + V_{HG} + V_{GF} + V_{FE}$

Or $V_{CE} = V_{CD} + V_{DA} + V_{AB} + V_{BH} + V_{HE}$ Or $V_{CE} = V_{CD} + V_{DA} + V_{AB} + V_{BH} + V_{HG} + V_{GF} + V_{FE}$

❖ $V_{CB} = -v_2 = -5i_1$

$V_{HE} = -v_5 = -5i_2$

$V_{BH} = 10$

❖ $V_{CB} + V_{BH} + V_{HE} = (V_C - V_B) + (V_B - V_H) + (V_H - V_E) = V_C - V_E = V_{CE}$

❖ $V_{AG} = V_{AB} + V_{BH} + V_{HG} = (V_A - V_B) + (V_B - V_H) + (V_H - V_G) = V_A - V_G = V_{AG}$

**Example 8:** Determine the branch currents in the network of **Figure (3.13)** when the value of each branch resistance is one ohm, using Kirchhoff's laws.

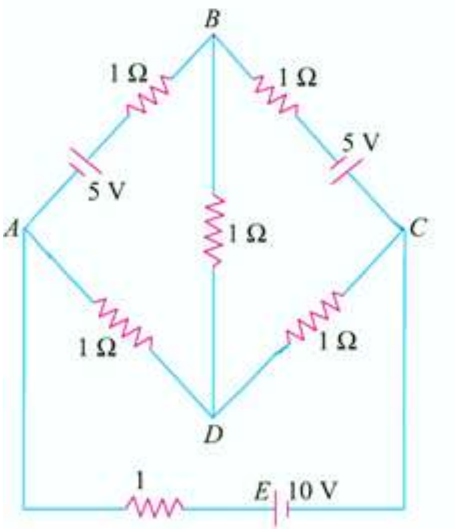



**Figure (3.13)**

**Solution:**

Indicate the currents direction in the network branches as shown in **Figure (3.14).**

Apply Kirchhoff's Second law to the closed circuit *ABDA*, we get

$5 - x - z + y = 0$ or $x - y + z = 5$ ...**(*i*)**

Similarly, circuit *BCDB* gives

$-(x - z) + 5 + (y + z) + z = 0$ or $x - y - 3z = 5$ ...**(*ii*)**

Lastly, from circuit *ADCEA*, we get

$-y - (y + z) + 10 - (x + y) = 0$ or $x + 3y + z = 10$ ...**(*iii*)**

From Eq. (*i*) and (*ii*), we get, $z =$ **0**

Substituting $z = 0$ either in Eq.(*i*) or (*ii*) & in Eq.(*iii*), we get

$x - y = 5$ ...**(*iv*)**

$x + 3y = 10$ ...**(*v*)**

Subtracting Eq. **(*v*)** from **(*iv*)**, we get

$-4y = -5$ or $y = 5/4 =$ **1.24 A**

Eq. (*iv*) gives $x = 25/4$ A = **6.25 A**

Current in branch *AB* = current in branch *BC* = **6.25 A**

Current in branch *BD* = **0**

current in branch *AD* = current in branch *DC* = **1.25 A**

current in branch *CEA* = 6.25 + 1.25 = **7.5 A.**

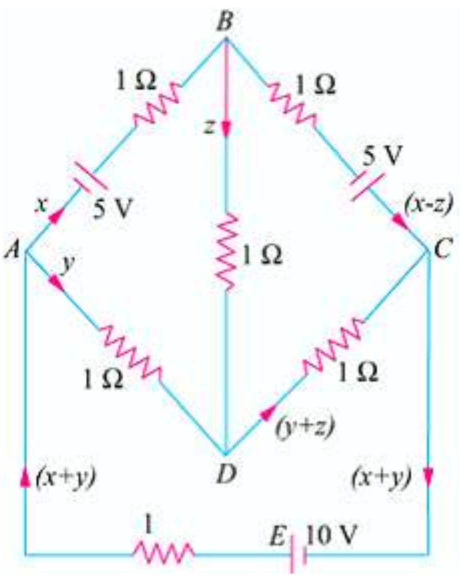



**Figure (3.14)**

**H.W.:** For **Example 7**, find the p.d. across AC.

**Example 9:** Determine the current x in the 4-Ω resistance of the circuit shown in **Figure (3.15)**, using Kirchhoff's laws.

**Solution:**

The given circuit is redrawn with assumed distribution of currents in **Figure (3.16)**. Applying KVL to different closed loops, we get

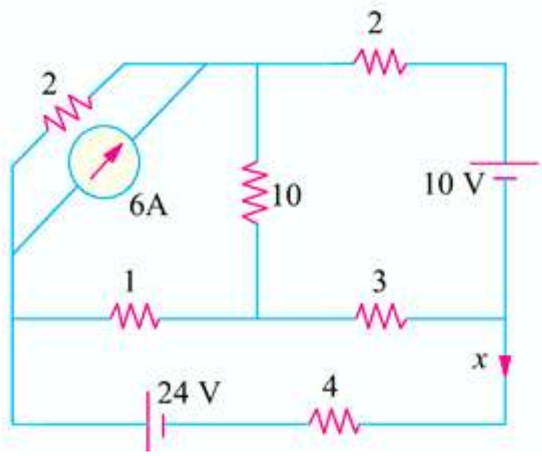


**Figure (3.15)**

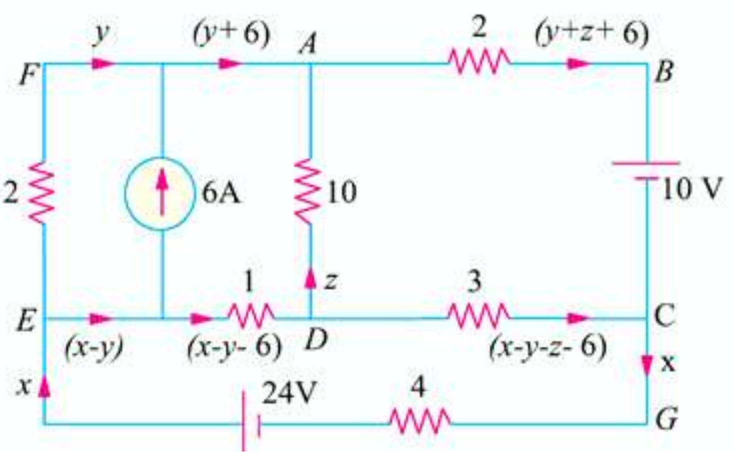


**Figure (3.16)**

**Circuit EFADE**

$$-2y + 10z + (x - y - 6) = 0$$

or $\quad x - 3y + 10z = 6 \qquad$ ...(*i*)

**Circuit ABCDA**

$$2(y + z + 6) - 10 + 3(x - y - z - 6) - 10z = 0$$

or $\quad 3x - 5y - 14z = 40 \qquad$ ...(*ii*)

**Circuit EDCGE**

$$-(x - y - 6) - 3(x - y - z - 6) - 4x + 24 = 0$$

or $\quad 8x - 4y - 3z = 48 \qquad$ ...(*iii*)

From above equations we get $x$ = **4.1 A**

**Example 10:** By applying Kirchhoff's current law, obtain the values of v, $i_1$ and $i_2$ in the circuit of **Figure (3.17)** which contains a voltage-dependent current source. Resistance values are in ohms.

**Solution:**

Applying KCL to node A of the circuit, we get

$$2 - i_1 + 4v - i_2 = 0$$

or $\quad i_1 + i_2 - 4v = 2$

Now, $\quad i_1 = \frac{v}{3} \quad$ and $\quad i_2 = \frac{v}{6}$

$\therefore \quad \frac{v}{3} + \frac{v}{6} - 4v = 2 \quad \Rightarrow \quad v = \frac{-4}{7}$ **V**

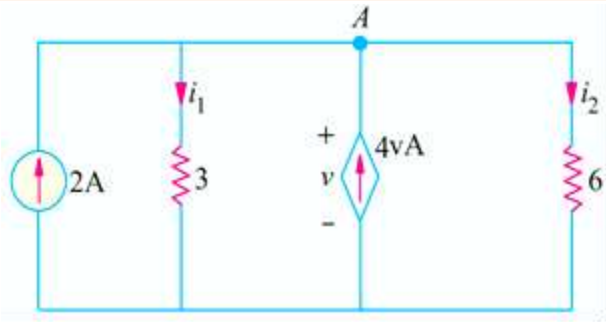



**Figure (3.17)**

$\therefore \quad i_1 = \frac{v}{3} = \frac{\left(\frac{-4}{7}\right)}{3} = \frac{-4}{21}$ **A** $\quad$ & $\quad i_2 = \frac{v}{6} = \frac{\left(\frac{-4}{7}\right)}{6} = \frac{-4}{42} = \frac{-2}{21}$ **A**

**Note**:- as the currents sign are negative then the actual direction of these currents are opposite to the direction indicated previously as shown in **Figure (3.18)**.

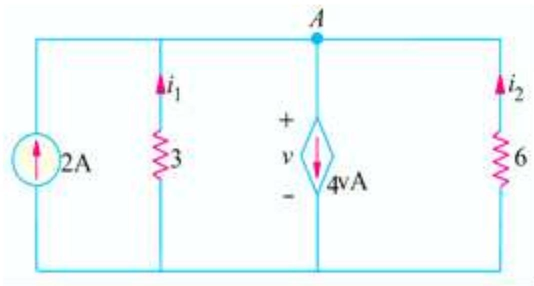



**Figure (3.18)**

& $\quad 4v = 4 \times \left(\frac{-4}{7}\right) = \frac{-16}{7}$ **V**

**Example 11:** Apply Kirchhoff's voltage law, to find the values of current i and the voltage drops $v_1$ and $v_2$ in the circuit of **Figure (3.19)** which contains a current-dependent voltage source. What is the voltage of the dependent source ? All resistance values are in ohms.

**Solution:**

Applying KVL to the circuit of **Figure (3.19)** and starting from point A, we get

$$-v_1 + 4i - v_2 + 6 = 0 \text{ or } v_1 - 4i + v_2 = 6$$

Now, $\quad v_1 = 2i$ and $v_2 = 4i$

$\therefore \quad 2i - 4i + 4i = 6$ or $i = 3A$

$\therefore \quad v_1 = 2 \times 3 = 6V$

and $\quad v_2 = 4 \times 3 =$ **12 V**

Voltage of the dependent source = $4i = 4 \times 4 =$ **12 V**

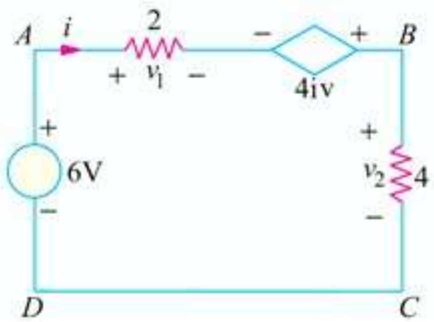



**Figure (3.19)**

**Example 12:** Find the current $i$ and also the power and voltage of the dependent source in **Figure (3.20)**. All resistances are in ohms.

**Solution:**

The two current sources can be combined into a single source of (8 − 6 = 2 A). The two parallel 4 Ω resistances when combined have a value of (2 Ω) which, being in series with the 10 Ω resistance, gives the branch resistance of (10 + 2 = 12 Ω). This 12 Ω resistance when combined with the other 12 Ω resistance gives a combination resistance of (6 Ω). The simplified circuit is shown in **Figure (3.21).**

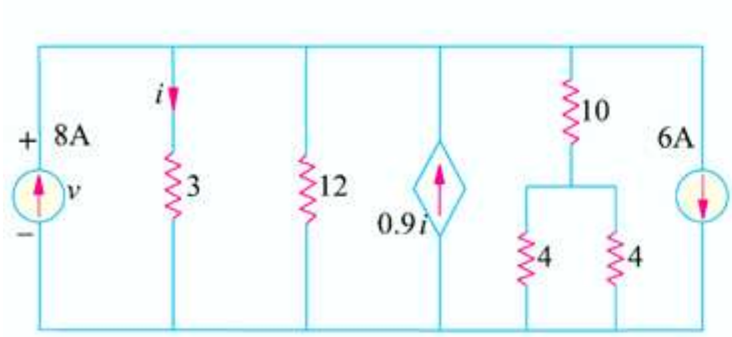



**Figure (3.20)**

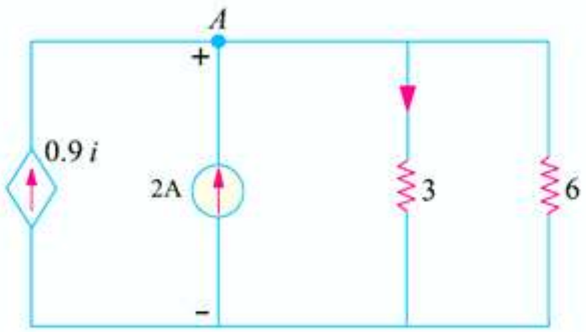



**Figure (3.21)**

Applying KCL to node A, we get

$0.9i + 2 - i - V/6 = 0$ or $0.6i = 12 - v$

Also $v = 3i$ ∴ $i = 10/3$ A.

Hence, v = **10 V.**

The power furnished by the current source = v × 0.9 $i$ = 10 × 0.9 (10/3) = **30 W.**

# Home Works

**H.W.(1):-** Find the values of currents $I_2$ and $I_4$ in the network of **Figure (3.22)**.

**[ Answer: $I_2 = 4$ A ; $I_4 = 5$ A]**

**H.W.(2):-** Use Kirchhoff's law, to find the values of voltages $V_1$ and $V_2$ in the network shown in **Figure (3.23)**.

**[Answer: $V_1 = 2$ V ; $V_2 = 5$ V]**

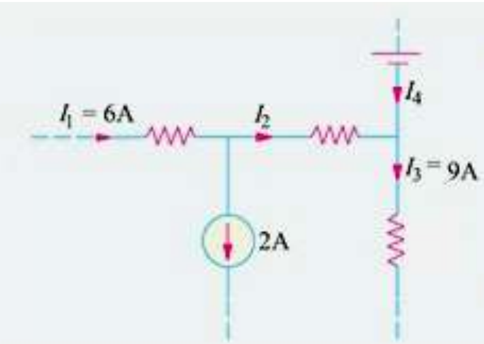



Figure (3.22)

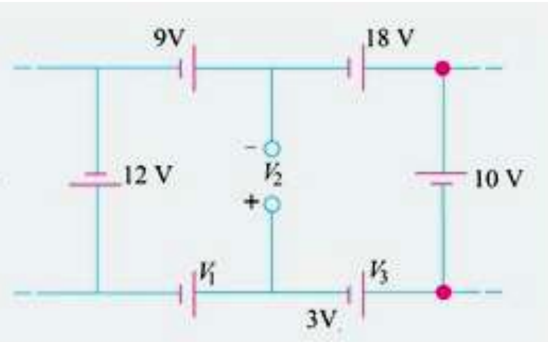



Figure (3.23)

**H.W.(3):-** In **Figure (3.24)**, the potential of point A is − 30 V. Using Kirchhoff's voltage law, find (a) value of V and (b) power dissipated by 5 Ω resistance. All resistances are in ohms.

**[100 V; 500 W]**

**H.W.(4):-** Using KVL and KCL, find the values of V and I in **Figure (3.25)**. All resistances are in ohms.

**[80 V; − 4 A]**

**H.W.(5):-** Using KCL, find the values $V_{AB}$, $I_1$, $I_2$ and $I_3$ in the circuit of **Figure (3.26)**. All resistances are in ohms.

**[$V_{AB} = 12$ V ; $I_1 = 2/3$ A; $I_2 = 1$ A; $I_3 = 4/3$ A]**

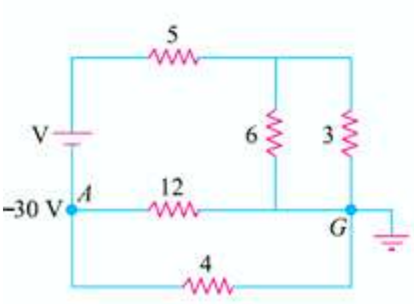



Figure (3.24)

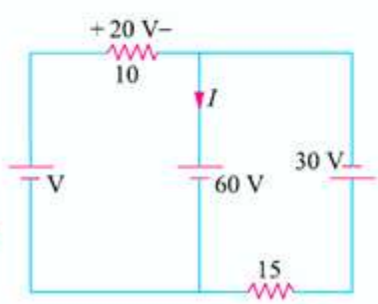



Figure (3.25)

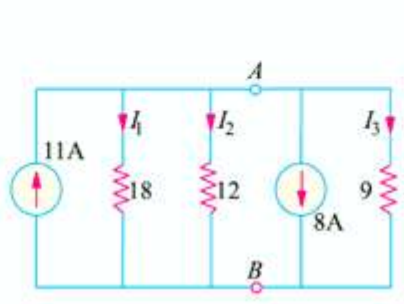



Figure (3.26)

**H.W.(6):-** Find the voltage of point A with respect to point B in the **Figure (3.27)**. Is it positive with respect to B ? **[potential of point A with respect to B is − 3.25 V]**

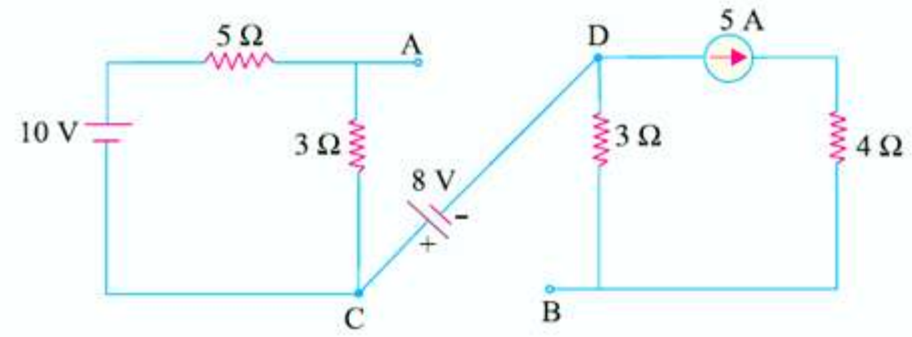



**Figure (3.27)**

**H.W.(7):-** Find the value of dependent voltage source & voltage $v_o$ in the **Figure (3.28)**. I

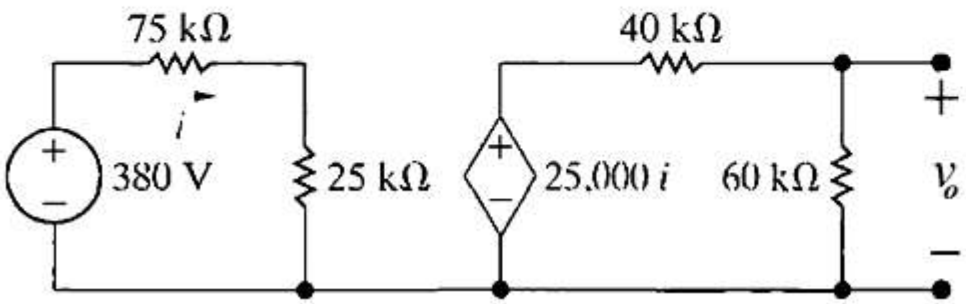



**Figure (3.28)**

Lecture (4)

# Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

## Steps to Determine Node Voltages:

1) Select a node as the reference node (it's voltage equal zero). Assign voltages $v_1$, $v_2$, ..., $v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.

2) Apply KCL to each of the n-1 non reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

3) Solve the resulting simultaneous equations to obtain the unknown node voltages.

The first step in nodal analysis is selecting a node as the ***reference*** or ***datum node***. The reference node is commonly called the *ground* since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in **Figure (4.1)**.

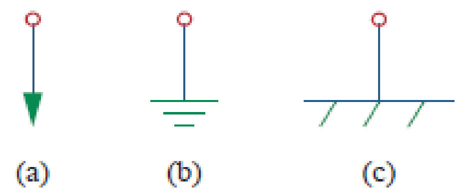



**Figure (4.1):** Common symbols for indicating a reference node

## 1- Nodal Analysis with Current Sources

**Note:** Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

$\therefore\ I_1 = \dfrac{v_o - v_1}{R_1}$, $I_2 = \dfrac{v_2 - v_1}{R_2}$, $i_x = \dfrac{v_2 - v_o}{R_3}$,

$i_5 = \dfrac{v_3 - v_2}{R_4}$ , $i_6 = \dfrac{v_4 - v_o}{R_5}$, $5i_x = \dfrac{v_1 - v_4}{R_6}$

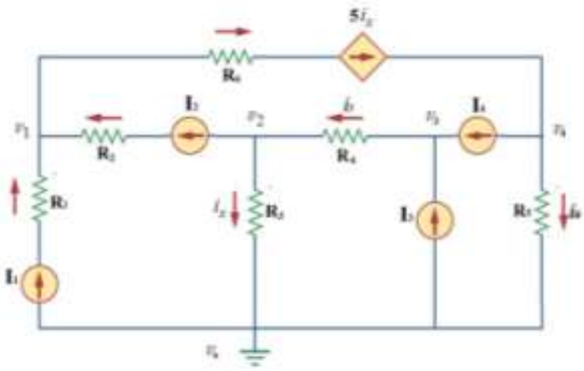


Figure (4.2)

Where $V_o = 0$

For node (1):

Apply KCL, $5i_x - I_1 - I_2 = 0$

$$\therefore \qquad i_x = \frac{I_1 + I_2}{5} \qquad \ldots(1)$$

For node (2):

Apply KCL, $I_2 + i_x - i_5 = 0$

$$I_2 + \frac{v_2}{R_3} - \frac{v_3 - v_2}{R_4} = 0 \qquad \ldots(2)$$

For node (3):

Apply KCL, $i_5 - I_3 - I_4 = 0$

$$\therefore \qquad i_5 = I_3 + I_4 \qquad \ldots(3)$$

For node (4):

Apply KCL, $i_6 + I_4 - 5i_x = 0$

$$\therefore \qquad \frac{v_4}{R_5} + I_4 - 5i_x = 0 \qquad \ldots(4)$$

Solving equations (1), (2), (3), and (4) to find the nodes voltages ($v_1, v_2, v_3, and\ v_4$) and hence find the currents in the circuit ($i_x$ , $i_5$ , and $i_6$).

## 2- Nodal Analysis with Voltage Sources

$\therefore i_1 = \frac{E_1 - v_1}{R_1}$, $i_2 = \frac{v_2 + E_2 - v_1}{R_2}$, $i_x = \frac{v_2 - v_o}{R_3}$, $i_5 = \frac{v_3 - v_2}{R_4}$, $i_3 = \frac{v_1 - 5i_x - v_4}{R_6}$, $i_4 = \frac{v_4}{R_5}$

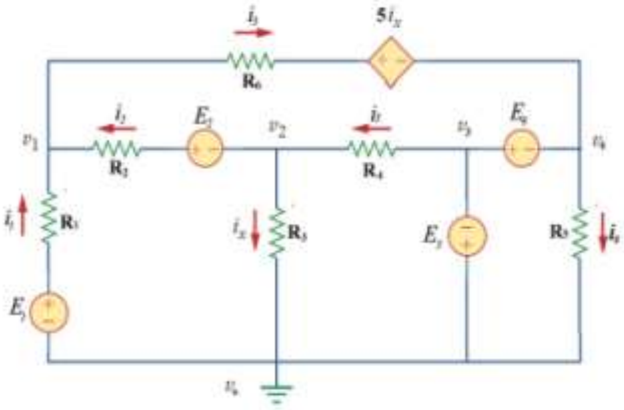


Figure (4.3)

For node (1):

Apply KCL, $-i_1 - i_2 + i_5 = 0$

$$\frac{v_1 - E_1}{R_1} + \frac{v_1 - E_2 - v_2}{R_2} + \frac{v_1 - 5i_x - v_4}{R_6} = 0$$

For node (2):

Apply KCL, $i_x + i_2 - i_5 = 0$

$$\frac{v_2}{R_3} + \frac{v_2 + E_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_4} = 0$$

For supernode (3) & (4) :

$V_3 + E_3 = 0,$ ⟹ $\therefore V_3 = E_3$

$V_3 - E_4 - V_4 = 0$ (Super node) ⟹ $\therefore V_3 = E_4 + V_4$

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non reference nodes and any elements connected in parallel with it.

**Example 1:** Determine the nod to reference voltages of the circuit shown in **Figure (4.4)**.

**Solution:**

For node 1:

$v_1 + 12 = 0$ ⟹ $\therefore v_1 = -12$ V

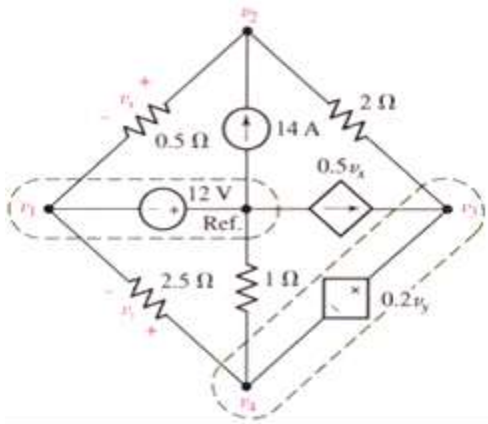


Figure (4.4)

For node (2):

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} - 14 = 0$$

$$\frac{v_2 + 12}{0.5} + \frac{v_2 - v_3}{2} = 14 \quad \ldots(1)$$

$v_x = v_2 - v_1$

For super node (3) & (4):

$v_3 - 0.2v_y - v_4 = 0$ …(2)

$\frac{v_3 - v_2}{2} - 0.5v_x + \frac{v_4 - v_1}{2.5} + \frac{v_4}{1} = 0$ …(3)

$v_4 - v_y - v_1 = 0$ …(4)

solving equations (1), (2), (3), and (4), ⇒ $v_1 = -12$ **V**, $v_2 = -4$ **V**, $v_3 = 0$ **V**, $v_4 = -2$ **V**

**Example 2:** Determine the current in (7Ω) & (1Ω) resistors using nodal analysis of the circuit shown in **Figure (4.5)**.

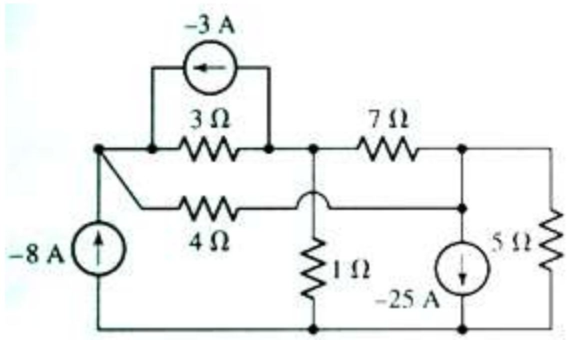


**Figure (4.5)**

**Solution:**

Redraw the circuit and indicate the nodes and reference node as shown in **Figure (4.6)**.

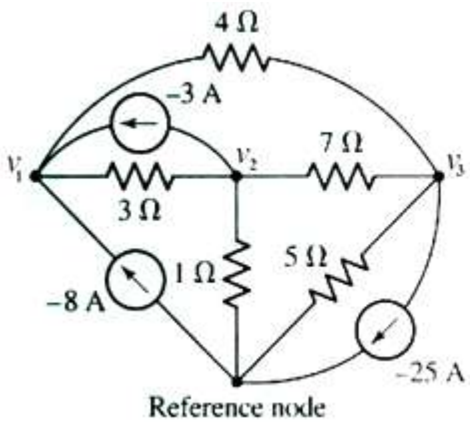


**Figure (4.6)**

At node (1):

$$\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$$

∴ $0.5833\, v_1 - 0.3333\, v_2 - 0.25\, v_3 = -11$ …(1)

At node (2):

$$\frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_2 - v_3}{7} - 3 = 0$$

∴ $-0.3333\, v_1 + 1.4762\, v_2 - 0.1429\, v_3 = 3$ …(2)

At node (3):

$$\frac{v_3}{5} + \frac{v_3 - v_2}{7} + \frac{v_3 - v_1}{4} - 25 = 0$$

∴ $-0.25\, v_1 - 0.1429\, v_2 + 0.5929\, v_3 = 25$ …(3)

Solving Equ.s (1), (2) & (3) we get, ⇒ $v_1 = 5.412$ **V**, $v_2 = 7.736$ **V** & $v_3 = 46.32$ **V**

Now, Current in (7Ω) is, $I_{7\Omega} = \frac{v_3 - v_2}{7} = \frac{46.32 - 7.736}{7} =$ **5.512 A**

Current in (1Ω) is, $I_{1\Omega} = \frac{v_2 - v_o}{1} = \frac{v_2 - 0}{1} = \frac{v_2}{1} = \frac{7.736}{1} =$ **7.736 A**

**Example 3:** Using nodal analysis, determine the current in ( $\frac{1}{3}$ S) of the circuit shown in **Figure (4.7)**.

**Solution:**

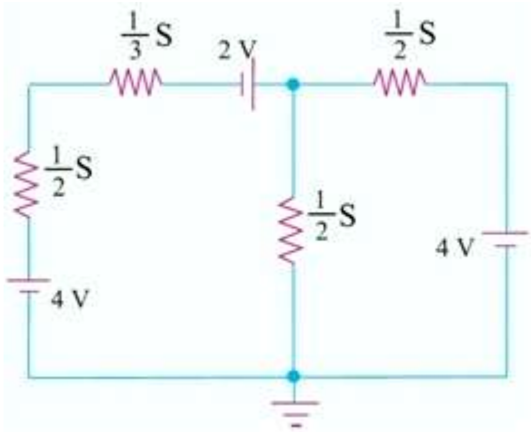



Figure (4.7)

**Note** that in this circuit a conductance values were given, so we convert it to a resistances and redraw the circuit as shown in **Figure (4.8)**.

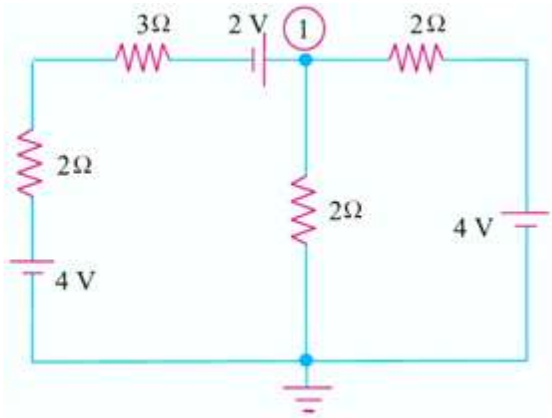



Figure (4.8)

At node (1):

$$\frac{v_1 - 2 - 4}{3+2} + \frac{v_1}{2} + \frac{v_1 - 4}{2} = 0$$

$\therefore v_1 =$ **2.67 V**

So the current through ( $\frac{1}{3}$ S) = current through (3Ω)

$$= \frac{v_1 - 2 - 4}{3+2}$$

$$= \frac{2.67 - 2 - 4}{3+2}$$

$= -$ **0.666 A**

**Example 4:** Determine the power supplied by the dependent source of the **Figure (4.9)** using nodal analysis.

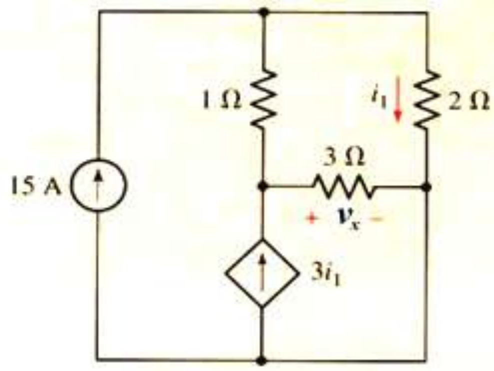



Figure (4.9)

**Solution:**

Redraw the circuit and indicate the nodes and reference node as shown in **Figure (4.10)**.

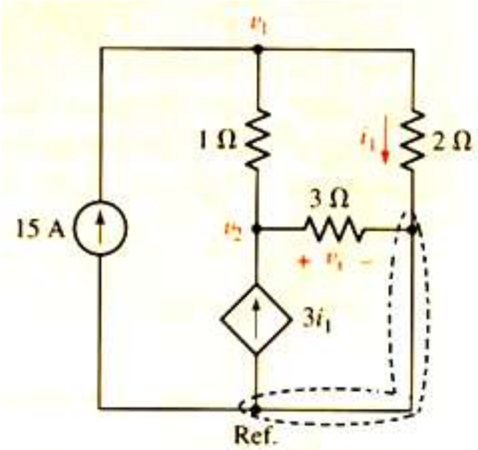



Figure (4.10)

At node (1):

$$\frac{v_1 - v_2}{1} + \frac{v_1}{2} - 15 = 0$$

$$\therefore 1.5\, v_1 - v_2 = 15 \qquad \ldots(1)$$

At node (2):

$$\frac{v_2 - v_1}{1} + \frac{v_2}{3} - 3i_1 = 0 \qquad \}\times 3$$

$$3\, v_2 - 3\, v_1 + v_2 - 9i_1 = 0$$

$$-3\, v_1 + 4v_2 - 9i_1 = 0, \quad i_1 = \frac{v_1}{2}$$

$$\therefore\ -3\, v_1 + 4v_2 - 9\left(\frac{v_1}{2}\right) = 0$$

$$\therefore -7.5\, v_1 + 4v_2 = 0 \qquad \ldots(2)$$

Solve Equ.s (1) & (2) we get,

$v_1 =$ **– 40 V** & $v_2 =$ **– 75 V** & $i_1 = \frac{v_1}{2} = \frac{-40}{2} =$ **– 20 A**

Now the power supplied by the dependent source is,

P = IV = $(3i_1)\, v_2 = (3\times(-20)) \times (-75) = 4500$ W = **4.5 kW**

**Example 5:** Determine the power supplied by the dependent source of the **Figure (4.11)** using nodal analysis.

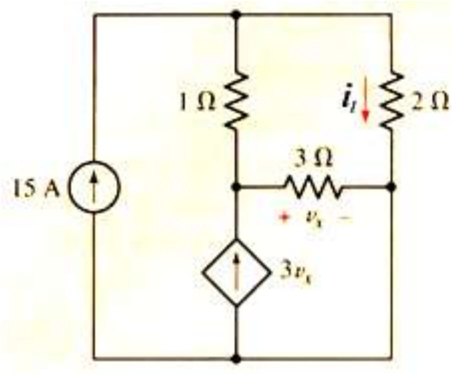



**Figure (4.11)**

**Solution:**

Redraw the circuit and indicate the nodes and reference node as shown in **Figure (4.12)**.

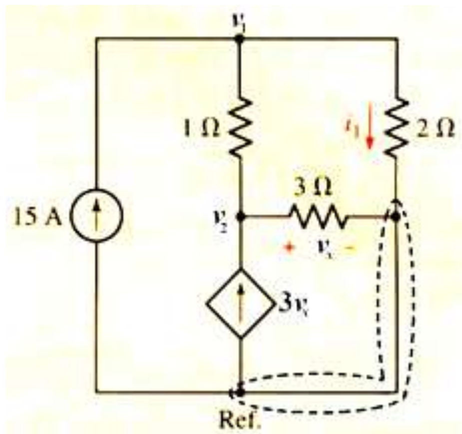



**Figure (4.12)**

<u>At node (1):</u>

$$\frac{v_1 - v_2}{1} + \frac{v_1}{2} - 15 = 0,$$

$$\therefore 1.5\, v_1 - v_2 = 15 \qquad \ldots(1)$$

<u>At node (2):</u>

$$\frac{v_2 - v_1}{1} + \frac{v_2}{3} - 3v_x = 0, \quad v_x = v_2$$

$$\therefore \frac{v_2 - v_1}{1} + \frac{v_2}{3} - 3v_2 = 0 \qquad \}\times 3$$

$$3\, v_2 - 3\, v_1 + v_2 - 9v_2 = 0$$

$$\therefore -3\, v_1 - 5v_2 = 0 \qquad \ldots(2)$$

Solve Equ.s (1) & (2) we get,
$v_1$ = **7.14 V** & $v_2 = v_x$ = **– 4.29 V**

Now the power supplied by the dependent source is,
P = IV = $(3v_x)\, v_x$ = (3×(– 4.29)) × (– 4.29) = **55.2 W**

# Home Works

**H.W.(1):-** Find $v_1$, $v_2$ and $v_3$ in the circuit of **Figure (4.13)** using nodal analysis.

**[ Answer: $v_1 = 3.043$ V, $v_2 = -6.956$ V & $v_3 = 0.6522$ V ]**

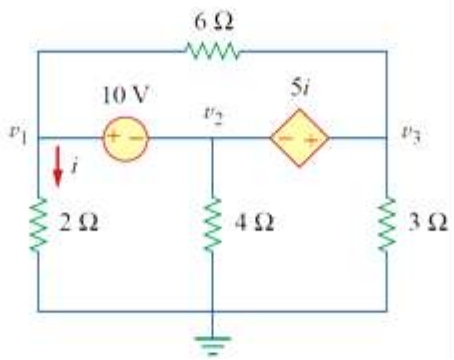



**Figure (4.13)**

**H.W.(2):-** Find $v_1$, $v_2$ and $v_3$ in the circuit of **Figure (4.14)** using nodal analysis.

**[ Answer: $v_1 = -40$ V, $v_2 = 57.14$ V & $v_3 = 200$ V ]**

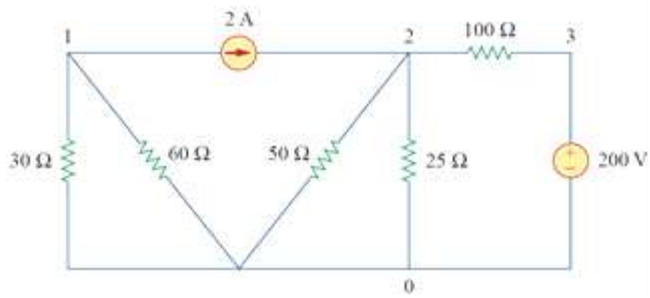



**Figure (4.14)**

**H.W.(2):-** Find $v_1$, $v_2$, $v_o$, $i_1$, $i_2$ & $i_3$ in the circuit of **Figure (4.15)** using nodal analysis.

**[ Answer: $v_1 = 2.67$V, $v_2 = 10.67$ V & $v_o = 10.67$ V, $i_1 = 1.34$A, $i_2 = 1.33$A, $i_3 = 2.67$A, $i_4 = 12$A, $i_5 = -8$A, $i_6 = 5.33$A,]**

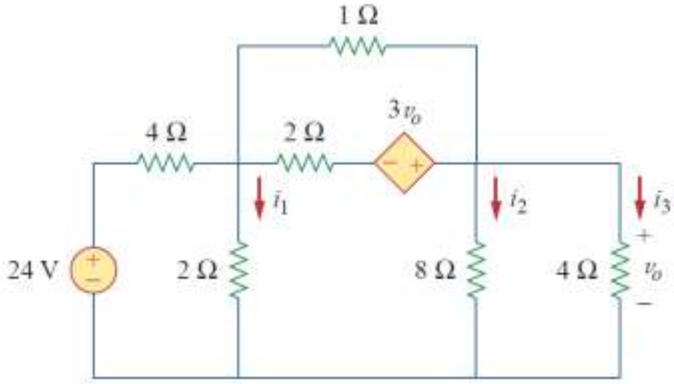



**Figure (4.15)**

Lecture (5)

# Mesh Analysis
# (Maxwell's Loop Curent Method)

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables instead of branch currents (as in Kirchhoff's laws). Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Mesh analysis applies KVL to find unknown currents.

A mesh is a loop that does not contain any other loop within it.

## Steps to Determine Mesh Currents:

1. Assign mesh currents to the *n* meshes.
2. Apply KVL to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting *n* simultaneous equations to get the mesh currents.

For the first loop:

$-V_1 + i_1R_1 + (i_1 - i_2)R_3 = 0$ …(1)

or

$i_1(R_1 + R_3) - i_2R_3 = V_1$ …(2)

For the second loop:

$V_2 + i_2R_2 + (i_2 - i_1)R_3 = 0$ …(3)

or

$-i_1 R_3 + i_2(R_2 + R_3) = -V_2$ …(4)

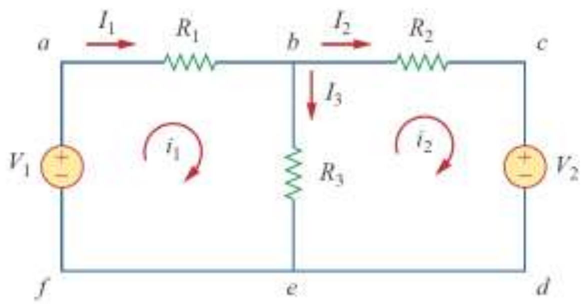



Figure (5.1)

Equations (2) & (4) can be solved either simultaneously or by matrix.

$$\begin{bmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

So $i_1 = \frac{\Delta_1}{\Delta}$ and $i_2 = \frac{\Delta_2}{\Delta}$

Where

$$\Delta = \begin{vmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} V_1 & -R_3 \\ -V_2 & R_2+R_3 \end{vmatrix}, \text{ and } \Delta_2 = \begin{vmatrix} R_1+R_3 & V_1 \\ -R_3 & -V_2 \end{vmatrix}$$

After finding the mesh current, $I_1 = i_1$, $I_3 = i_1 - i_2$, and $I_2 = i_2$

**Example 1:** For the circuit in **Figure (5.2)**, find the branch currents using mesh analysis.

**Solution:**

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \quad \textbf{(1)}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \quad \textbf{(2)}$$

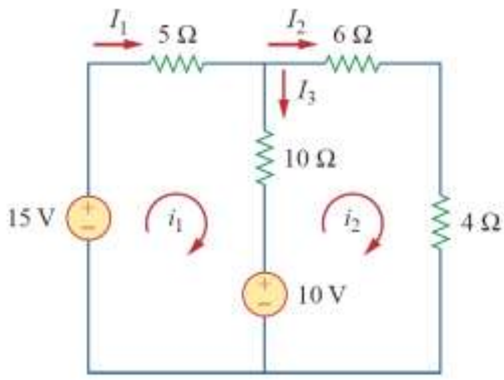



**Figure (5.2)**

**METHOD 1** Using the substitution method, we substitute Eq. (2) into Eq. (1), and write

$$6i_2 - 3 - 2i_2 = 1 \quad \Rightarrow \quad i_2 = 1\text{ A}$$

From Eq. (2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1\text{ A}, \qquad I_2 = i_2 = 1\text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

**METHOD 2** To use Cramer's rule, we cast Eqs. (1) and (2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1\text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1\text{ A}$$

as before.

**Example 2:** Use mesh analysis to find the current $I_o$ in the circuit of **Figure (5.3)**.

**Solution:**

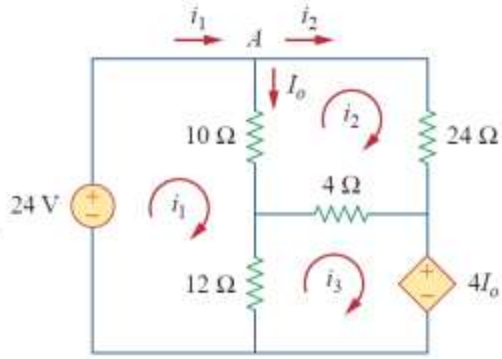



**Figure (5.3)**

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (2)$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

In matrix form, Eqs. (1) to (3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix} \begin{matrix} \\ \\ \\ + \\ + \\ + \end{matrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

## Mesh Analysis with Current Sources

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

■ **CASE 1** when a current source exists only in one mesh: Consider the circuit in **Figure (5.4)** , for example. We set $i_2$ = -5 A and write a mesh equation for the other mesh in the usual way; that is,

$-10 + 4i_1 + 6(i_1 - i_2) = 0$, and $i_2 = -5$ A

$\therefore i_1 = -2$ A

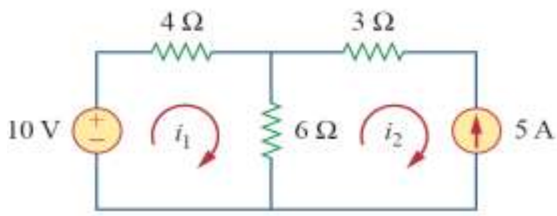



**Figure (5.4)**

■ **CASE 2** When a current source exists between two meshes: Consider the circuit in **Figure (5.5)(a)** , for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in **Figure (5.5)(b)**. Thus,

| |
|---|
| A supermesh results when two meshes have a (dependent or independent) current source in common. |

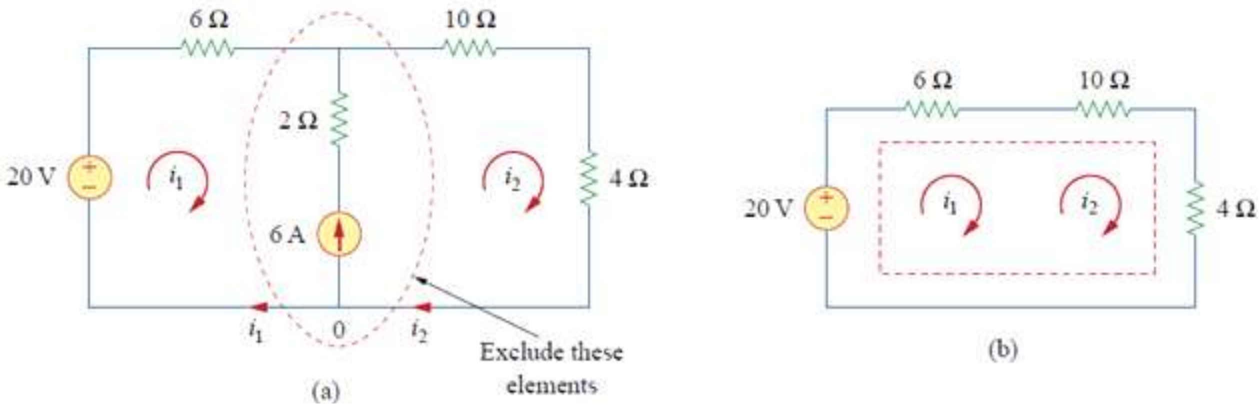



**Figure (5.5)**

From **Figure (5.5)(a),** Applying KCL to node 0 in,

$i_2 = 6 + i_1$ A …(1)

So from **Figure (5.5)(b),** the mesh equation is,

$-20 + 6i_1 + 10i_2 + 4i_2 = 0$

$6i_1 + 14i_2 = 20$ …(2)

Solving equation (1) & (2), we get,

$i_1 = -3.2$ A and $i_2 = -2.8$ A

**Example 3:** Use mesh analysis to determine $i_1$, $i_2$ & $i_3$ for the circuit in **Figure (5.6)**.

**Solution:**

**Note:** mesh (1) & mesh (2) are supermesh, because there exists a current source between them.

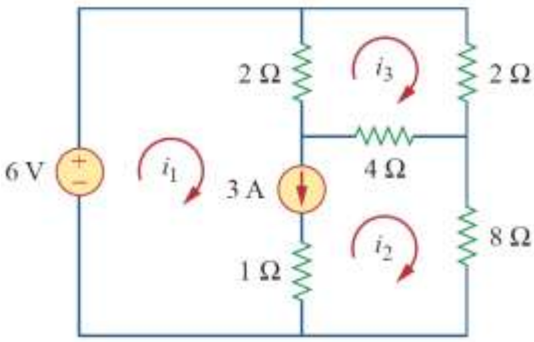



**Figure (5.6)**

For supermesh (1) & (2):

1) Find the relation among $i_1$, $i_2$ and the current source, so take node (s) in **Figure (5.7)** and apply KCL to find this relation.

$i_1 = i_2 + 3$ …(1)

2) Eliminate the current source branch as in **Figure (5.7)** and write the supermesh equation as follows,

$v_1 = 2(i_1 - i_3)$, $v_2 = 4(i_2 - i_3)$, $v_3 = 8i_2$ & $v_4 = 2i_3$

$-6 + v_1 + v_2 + v_3 = 0$

$-6 + 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2 = 0$

$2i_1 + 12i_2 - 6i_3 = 6$ } ÷ 2

$\therefore\ i_1 + 6i_2 - 3i_3 = 3$ …(2)

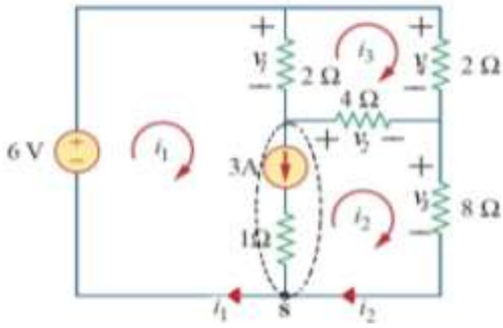



**Figure (5.7)**

For mesh (3):

$v_4 - v_2 - v_1 = 0$

$2i_3 - 4(i_2 - i_3) - 2(i_1 - i_3) = 0$ } ÷ −2

$-i_3 + 2(i_2 - i_3) + (i_1 - i_3) = 0$

$\therefore\ i_1 + 2i_2 - 4i_3 = 0$ …(3)

Solve equ.s (1), (2) & (3) we get,

$i_1 = 3.474$ A, $i_2 = 0.4737$ A, $i_3 = 1.1052$ A.

**Example 4:** Use mesh analysis to determine the current flowing in the (4Ω) resistor for the circuit shown in **Figure (5.11)**.

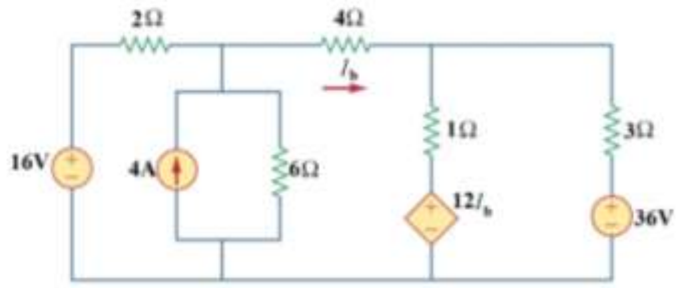



**Figure (5.11)**

**Solution:**

Redraw the circuit as shown below & indicate the voltage polarities on each element in the circuit.

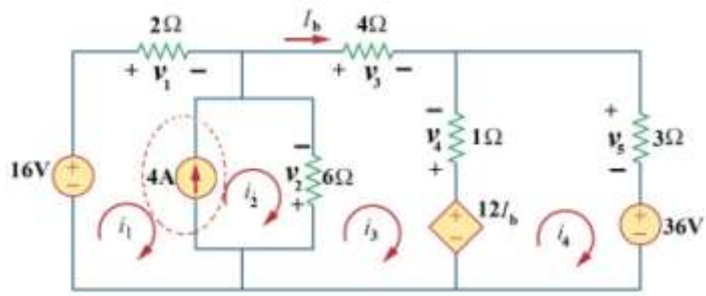



**Figure (5.12)**

<u>For supermesh (1) & (2):</u>

$i_2 = i_1 + 4$ …(1)

$v_1 = 2i_1,\ v_2 = -6(i_2 - i_3)$

$-16 + v_1 - v_2 = 0$

$-16 + 2i_1 + 6(i_2 - i_3) = 0$

$\therefore\ 2i_1 + 6i_2 - 6i_3 = 16$ … (2)

Substitute Equ.(1) in (2), we get,

$2i_1 + 6(i_1 + 4) - 6i_3 = 16$

$\therefore\ 8i_1 - 6i_3 = -8$ … (3)

<u>For mesh (3):</u>

$v_3 = 4i_3,\ v_4 = -1(i_3 - i_4)$

$12I_b - v_4 + v_3 + v_2 = 0$

$12I_b + 1(i_3 - i_4) + 4i_3 - 6(i_2 - i_3) = 0$

It is clearly that, $\boxed{I_b = i_3}$

$\therefore\ -6i_2 + 23i_3 - i_4 = 0$ …(4)

Substitute Equ.(1) in (4), we get,

$\therefore\ -6i_1 + 23i_3 - i_4 = 24$ …(5)

<u>For mesh (4):</u>

$v_5 = 3i_4$

$36 - 12I_b + v_4 + v_5 = 0$

$36 - 12\,i_3 + v_4 + v_5 = 0$

$36 - 12\,i_3 - 1(i_3 - i_4) + 3i_4 = 0$

$\therefore\ 13i_3 - 4i_4 = 36$ …(6)

Solve Equ.s (3), (5) & (6) we get,

$$\begin{bmatrix} 8 & -6 & 0 \\ -6 & 23 & -1 \\ 0 & 13 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -8 \\ 24 \\ 36 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 & -6 & 0 \\ -6 & 23 & -1 \\ 0 & 13 & -4 \end{bmatrix}^{-1} \begin{bmatrix} -8 \\ 24 \\ 36 \end{bmatrix}$$

$$\therefore \begin{bmatrix} i_1 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -0.557 \\ 0.59 \\ -7.082 \end{bmatrix}$$

$\therefore I_b = i_3 = 0.59$ A

**Note :-** mesh paths can take different ways as explained in **Examples (6), (7) & (8)**, but note that each element in the circuit must carry one loop or more.

**Example 5:** Use mesh analysis to determine the branch currents $I_1$, $I_2$ & $I_3$ for the circuit shown in **Figure (5.13)**.

**Solution:**

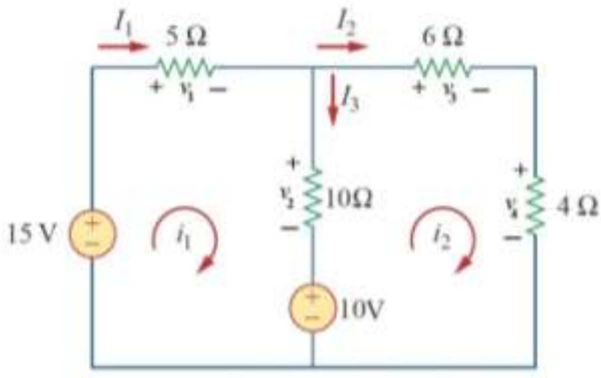



**Figure (5.13)**

For mesh (1):

$-15 + v_1 + v_2 + 10 = 0,\ \ v_1 = 5i_1\ \&\ \ v_2 = 10(i_1 - i_2)$

$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

$15i_1 - 10i_2 = 5 \quad \} \div 5$

$\therefore 3i_1 - 2i_2 = 1 \qquad \ldots(1)$

For mesh (2):

$v_3 + v_4 - v_2 - 10 = 0, \quad v_3 = 6i_2\ \&\ \ v_4 = 4\, i_2$

$6i_2 + 4\, i_2 - 10(i_1 - i_2) - 10 = 0$

$-10i_1 + 20i_2 = 10 \quad \} \div 10$

$\therefore -i_1 + 2i_2 = 1 \qquad \ldots(2)$

Solve Equ.s (1) & (2) simultaneously or by Grammer's rule or by matrix inversion method to find mesh currents,

$\therefore i_1 = 1\text{A},\ i_2 = 1\text{A}$

$I_1 = i_1 = 1\text{A}$, $\quad I_2 = i_2 = 1\text{A} \quad \& \quad I_3 = i_1 - i_2 = 0\text{A}$

**Example 6:** Repeat Example (6) for the mesh indicated in **Figure (5.14)**.

**Solution:**

For mesh (1):

$-15 + v_1 + v_2 + 10 = 0, \quad v_1 = 5i_1 \ \& \ v_2 = 10(i_1 + i_2)$

$-15 + 5i_1 + 10(i_1 + i_2) + 10 = 0$

$15i_1 + 10i_2 = 5 \quad \} \div 5$

$\therefore 3i_1 + 2i_2 = 1 \quad \ldots(1)$

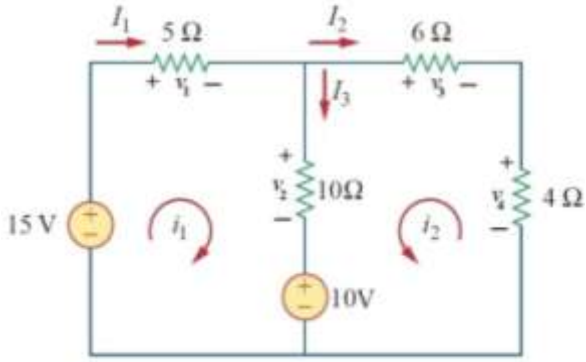



Figure (5.14)

For mesh (2):

$-v_3 - v_4 + v_2 + 10 = 0, \quad v_3 = -6i_2 \ \& \ v_4 = -4\, i_2$

$6i_2 + 4\, i_2 + 10(i_1 + i_2) + 10 = 0$

$10i_1 + 20i_2 = -10 \quad \} \div 10$

$\therefore i_1 + 2i_2 = -1 \quad \ldots(2)$

Solve Equ.s (1) & (2) simultaneously or by Grammer's rule or by matrix inversion method to find mesh currents,

$\therefore i_1 = 1\text{A},\ i_2 = -1\text{A}$

$I_1 = i_1 = 1\text{A},$ $\quad I_2 = -i_2 = 1\text{A}$ $\quad \&$ $\quad I_3 = i_1 + i_2 = 0\text{A}$

**Note:-** the elements currents must be same although the mesh paths are different but the mesh currents may be different which depend on the mesh paths.

**Example 7:** Repeat Example (6) for the mesh indicated in **Figure (5.15)**.

**Solution:**

For mesh (1):

$-15 + v_1 + v_2 + 10 = 0$

$v_1 = 5(i_1 + i_2) \ \& \ v_2 = 10i_1$

$-15 + 5(i_1 + i_2) + 10i_1 + 10 = 0$

$15i_1 + 5i_2 = 5 \quad \} \div 5$

$\therefore 3i_1 + i_2 = 1 \quad \ldots(1)$

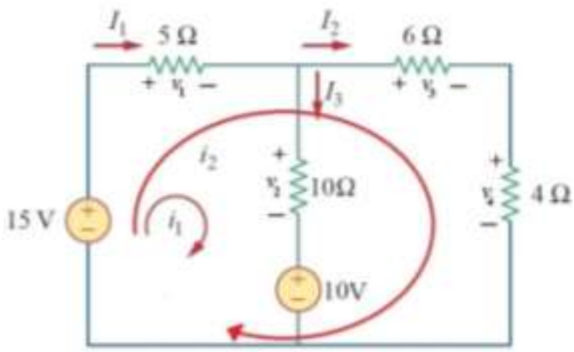



Figure (5.15)

For mesh (2):

$-15 + v_1 + v_3 + v_4 = 0$

$v_3 = 6i_2 \ \& \ v_4 = 4\, i_2$

$-15 + 5(i_1 + i_2) + 6i_2 + 4\, i_2 = 0$

$5i_1 + 15i_2 = 15 \quad \} \div 5$

$\therefore i_1 + 3i_2 = 3 \quad \ldots(2)$

Solve Equ.s (1) & (2) simultaneously or by Grammer's rule or by matrix inversion method to find mesh currents,

$\therefore i_1 = 0\text{A},\ i_2 = 1\text{A}$

$I_1 = i_1 + i_2 = 1\text{A},$ $\quad I_2 = i_2 = 1\text{A}$ $\quad \&$ $\quad I_3 = i_1 = 0\text{A}$

Same results for elements currents and different mesh currents.

# Home Works

**H.W.(1):** Use mesh analysis to determine the branch currents for the circuit shown in **Figure (5.16)**.

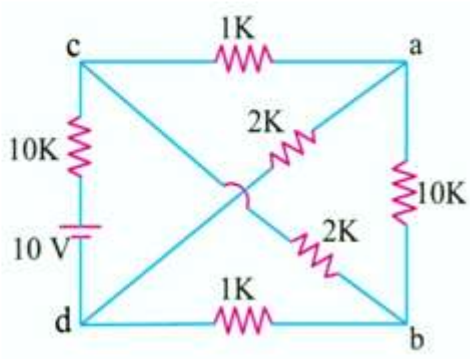



**Figure (5.16)**

Lecture (6)

# Superposition

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis.
Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the ***superposition***.
The idea of superposition rests on the linearity property.

> The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

## Steps to Apply Superposition Principle:

1) Turn off all independent sources (where voltage sources replaced by short circuit and current sources replaced by open circuit) except one source. Find the output (voltage or current) due to that active source using the techniques discussed previously such as *Ohm's law*, *Kirchhoff's laws*, *Source transformations*, *Mesh current* and *nodal Analysis*.

**Note:** Dependent sources are left intact (don't turn off) because they are controlled by circuit variables.

2) Repeat step 1 for each of the other independent sources.

3) Find the total contribution by adding algebraically all the contributions due to the independent sources.

However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.
Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

**Example 1:** Using the superposition theorem, find $v_o$ in the circuit of **Figure (6.1)** shown below.

**Solution:**

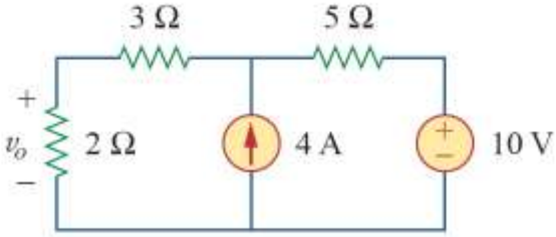



**Figure (6.1)**

1) Let the voltage source of (10 V) acting through the circuit and turn off the independent current source by replacing it by an open circuit and redraw the circuit as shown below.

The total current $I_{total} = \dfrac{V}{R_{total}} = \dfrac{V}{R_1 + R_2 + R_3}$

$$= \frac{10}{5+3+2} = 1\ \text{A}$$

$\therefore\ v_o' = 2 \times I_{total} = 2 \times 1 = \boxed{2\ \text{V}}$

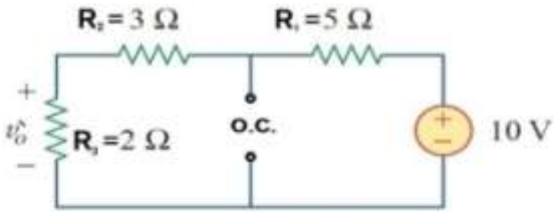



**Figure (6.2)**

2)Let the current source of (4 A) acting through the circuit and turn off the independent voltage source by replacing it by short circuit and redraw the circuit as shown below.

Use current division rule,

$I_{R3} = I_s \times \dfrac{R_1}{R_1 + (R_2 + R_3)}$

$$= 4 \times \frac{5}{5+(3+2)} = 2\ \text{A}$$

$\therefore\ v_o'' = 2 \times I_{R3} = 2 \times 2 = 4\ \text{V}$

$\therefore\ v_o = v_o' + v_o'' = 2 + 4 = \boxed{6\ \text{V}}$

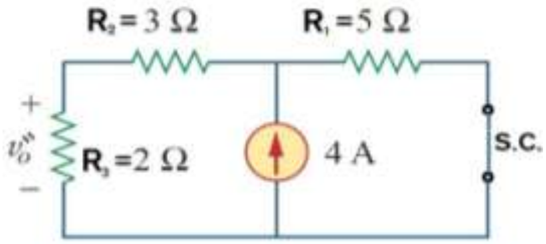



**Figure (6.3)**

**Example 2:** Use superposition to find $v_x$ in the circuit of **Figure (6.4)** shown below.

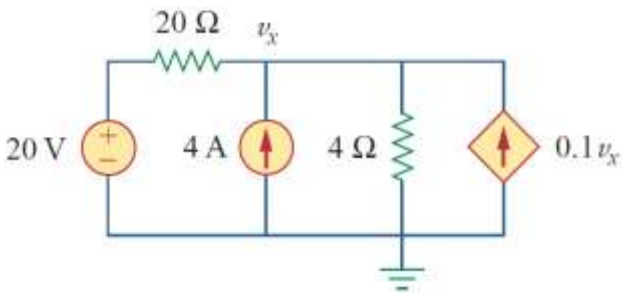



**Figure (6.4)**

**Solution:**

1)Let the voltage source of (20 V) acting through the circuit and turn off the independent current source by replacing it by an open circuit and redraw the circuit as shown below.

Use nodal analysis to find $v_x'$.

$$\frac{v_x' - 20}{20} + \frac{v_x'}{4} - 0.1v_x' = 0$$

$$\therefore v_x' = 5\text{ V}$$

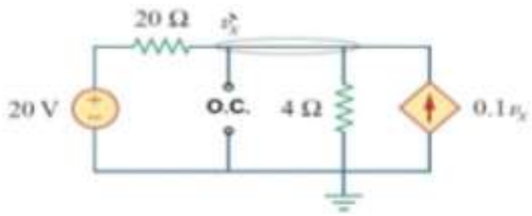



**Figure (6.5)**

2)Let the current source of (4 A) acting through the circuit and turn off the independent voltage source by replacing it by short circuit and redraw the circuit as shown below.

Use nodal analysis to find $v_x''$.

$$\frac{v_x''}{20} - 4 + \frac{v_x''}{4} - 0.1v_x'' = 0$$

$$\therefore v_x'' = 20\text{ V}$$

$$\therefore v_x = v_x' + v_x'' = 5 + 20 = 25\text{ V}$$

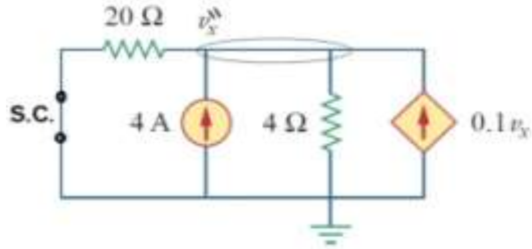



**Figure (6.6)**

**Alternative solution:**

1)Let the voltage source of (20 V) acting through the circuit and turn off the independent current source by replacing it by an open circuit.

Use source transformation and transform the voltage source of (20 V) to current source and redraw the circuit as shown in **Figure (6.7)**.

$$I_s = \frac{V_s}{20} = \frac{20}{20} = 1\text{ A}$$

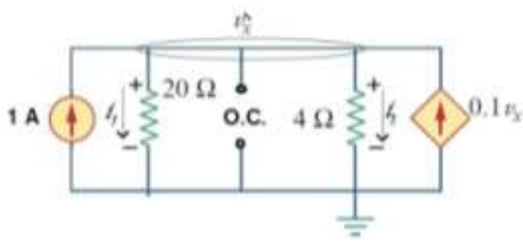



**Figure (6.7)**

Apply KCL,

$$1 + 0.1v_x' = i_1 + i_2\ ,\ i_1 = \frac{v_x'}{20} \quad\text{and}\quad i_2 = \frac{v_x'}{4}$$

$$\therefore 1 + 0.1v_x' = \frac{v_x'}{20} + \frac{v_x'}{4}\ ,$$

$$\therefore v_x' = 5\text{ V}$$

2)Let the current source of (4 A) acting through the circuit and turn off the independent voltage source by replacing it by short circuit and redraw the circuit as shown below.

Use source transformation and transform the Current sources of (4 and $0.1v_x''$) to current source and redraw the circuit as shown in figure.

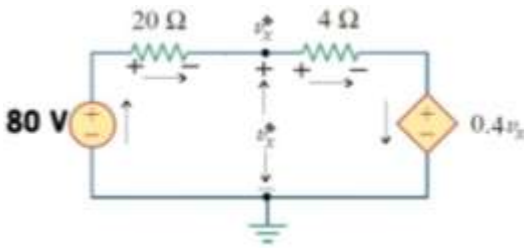



Figure (6.8)

$V_s = I_s \times R_{se}$

$= 4 \times 20 = 80$ V (for independent current source)

$= 0.1v_x'' \times 4 = 0.4v_x''$ V (for dependent current source)

Now apply KVL,

$-80 + 24i + 0.4v_x'' = 0$

$\therefore i = \frac{80 - 0.4v_x''}{24}$ …(1)

$-v_x'' + 4i + 0.4v_x'' = 0$ …(2)

Substitute equ. (1) in equ.(2), we get,

$-v_x'' + 4(\frac{80 - 0.4v_x''}{24}) + 0.4v_x'' = 0$

$\therefore v_x'' = 20$ V

$\therefore v_x = v_x' + v_x'' = 5 + 20 = 25$ V

**H.W/** Solve the above example by using Mesh Analysis.

**Lecture (7)**

# Thevenin's Theorem

**Thevenin's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $V_{Th}$ in series with a resistor $R_{Th}$, where $V_{Th}$ is the open-circuit voltage at the terminals and $R_{Th}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.

## How to find Thevenin's equivalent circuit

1) Remove $R_L$ from the circuit terminals *A* and *B* and redraw the circuit. Obviously, the terminals have become open-circuited.
2) Calculate the open-circuit voltage $V_{oc}$ which appears across terminals *A* and *B* when they are open *i.e.* when $R_L$ is removed. It is also called 'Thevenin voltage' $V_{Th}$.
3) Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit. It is also called Thevenin resistance $R_{Th}$.
4) Replace the entire network by a single Thevenin source, whose voltage is $V_{Th}$ or $V_{oc}$ and whose internal resistance is $R_{Th}$.
5) Connect $R_L$ back to its terminals from where it was previously removed.
6) Finally, calculate the current flowing through $R_L$ by using the equation,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{oc}}{R_{Th} + R_L} \quad , \quad V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

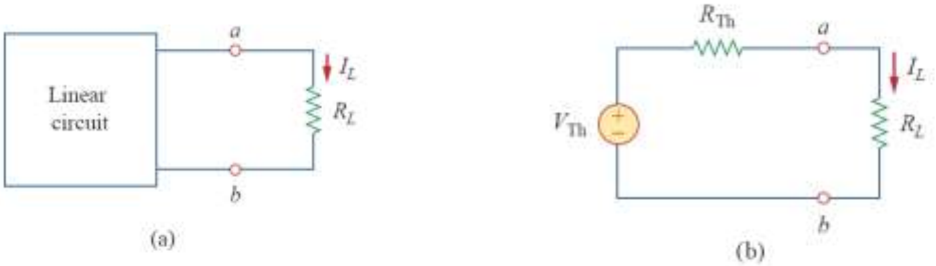



**Figure (7.1)**: A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

## How to find Thevenin's equivalent resistance ($R_{Th}$)

■ **CASE 1 If the network has no dependent sources**, we turn off all independent sources. $R_{Th}$ is the input resistance of the network looking between terminals a and b, as shown in **Figure (7.2-b).**

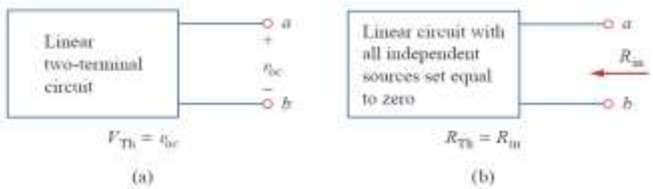



**Figure (7.2) :** Finding $V_{Th}$ and $R_{Th}$.

■ **CASE 2 If the network contains both dependent and independent sources,** we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables.

We apply a voltage source $v_o$ at terminals a and b and determine the resulting current. Then $R_{Th} = \frac{v_o}{i_o}$ , as shown in **Figure (7.3-a).** Alternatively, we may insert a current source $i_o$ at terminals a-b as shown in **Figure (7.3-b)** and find the terminal voltage $v_o$ Again $R_{Th} = \frac{v_o}{i_o}$. Either of the two approaches will give the same result. In either approach we may assume any value of $v_o$ and $i_o$ For example, we may use $v_o$ =1 V or $i_o$ =1 A, or even use unspecified values of $v_o$ or $i_o$

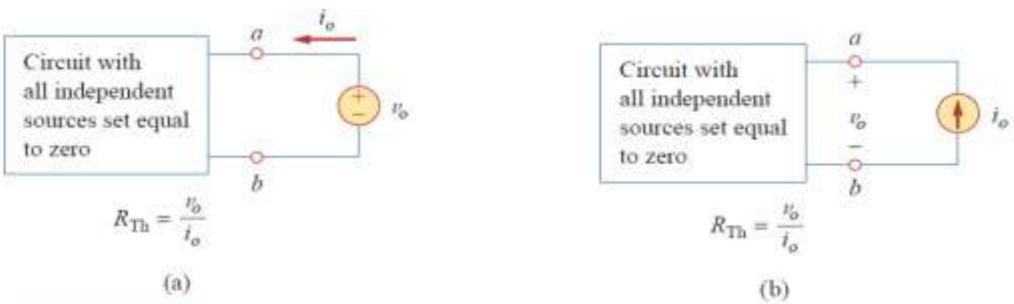



**Figure (7.3):** Finding $R_{Th}$ when circuit has dependent sources.

It often occurs that $R_{Th}$ takes a negative value. In this case, the negative resistance ($v = -iR$ ) implies that the circuit is supplying power.

**Alternative method:** (this method can applied for circuits doesn't have dependent sources and for circuits have dependent sources)

**(i)** The open-circuit voltage $V_{oc}$ is determined as usual with the sources activated.

**(ii)** A short-circuit is applied across the terminals *a* and *b* and the value of short-circuit current $i_{sc}$ is found as usual.

**(iii)** Thevenin resistance $R_{Th} = \frac{V_{oc}}{i_{sc}}$. It is the same procedure as adopted for Norton's theorem.

**Example 1:** Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit of Figure shown. Then find *I*.

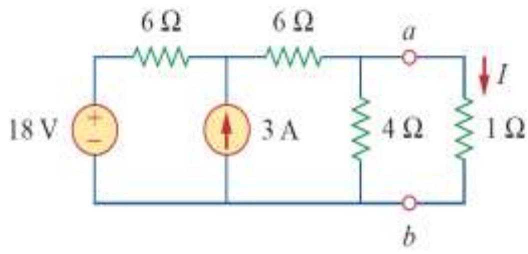



Figure (7.4)

**Solution:**

- Remove the branch where the required current (I) flow through it (the branch RL=1 Ω).

- To find $R_{Th}$ , turn-off all independent sources i.e. (replace voltage sources by S.C. and replace current sources by O.S.) as shown in figure below.

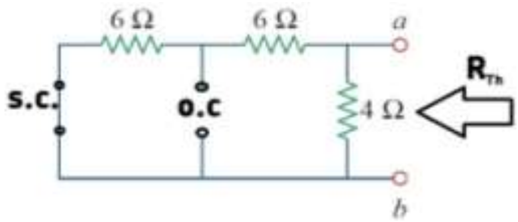



Figure (7.5)

$$R_{Th} = (6+6)||4 = \frac{12\times4}{12+4} = \boxed{3\Omega}$$

- To find $V_{Th}$ , use any method discussed previously to find voltage between terminals a & b, i.e. ($V_{oc}$ or $V_{Th}$).

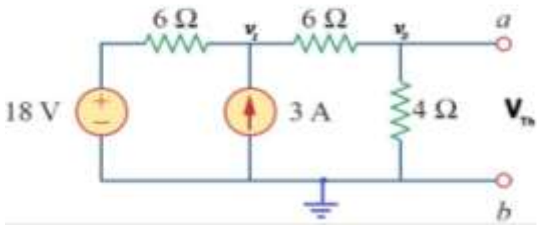



Figure (7.6)

**Use nodal analysis,**

For node (1),

$$\frac{V_1-18}{6} - 3 + \frac{V_1-V_2}{6} = 0$$

$$\boxed{2V_1 - V_2 = 36} \qquad \ldots(1)$$

For node (2),

$$\frac{V_2-V_1}{6} + \frac{V_2}{4} = 0$$

$$\boxed{2V_1 - 5V_2 = 0} \qquad \ldots(2)$$

Solve Equ.s (1) & (2), we get,

$$\boxed{V_2 = 9\text{ V} = V_{Th}}$$

- Connect the equivalent Thevenin's circuit as shown in **Figure (7.6)**,

$$I_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{9}{3+1} = \boxed{2.25\ A}$$

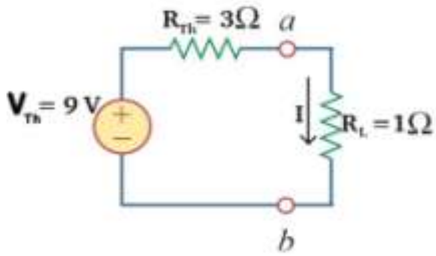



**Figure (7.6)**

**H.W.:-** Find $V_{Th}$ for example (1) by using Mesh analysis.

**Example 2:** Find the Thevenin equivalent circuit of the circuit shown in Figure to the left of the terminals.

**Solution:**

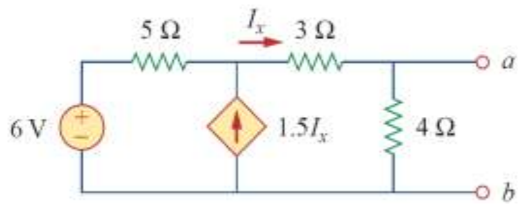



**Figure (7.6)**

- The circuit have dependent source, then to find $R_{Th}$, turn-off all independent sources and apply a voltage source ($v_o$=1 V) across terminals a & b, then find the source current ($i_o$) as shown in figure below.

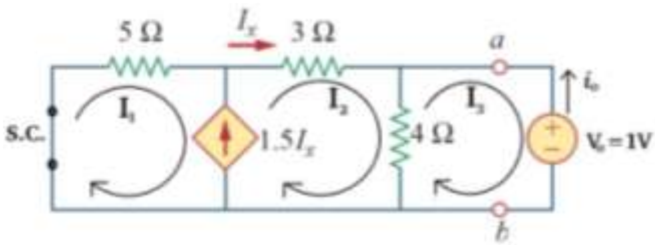



**Figure (7.7)**

**Use Mesh analysis,**

For super mesh (1) & (2),

$\boxed{I_2 = I_1 + 1.5I_x}$ …(1)

& $\boxed{I_x = I_2}$ …(2)

Substitute equation (2) in (1), we get,

$\boxed{I_1 = -0.5I_2}$ …(3)

$5I_1 + 3I_2 + 4(I_2 - I_3) = 0$

$\therefore \boxed{5I_1 + 7I_2 - 4I_3 = 0}$ …(4)

For mesh (3)

$4I_2 - 4I_3 = 1$

$\therefore$ $\boxed{I_2 - I_3 = 0.25}$ …(5)

Solve Equ.s (3), (4), & (5), we get,

$\boxed{I_2 = -2A = I_x}$ , $\boxed{I_3 = -2.25\ A = -i_o}$ ⟹ $\boxed{i_o = 2.25\ A}$

$\therefore R_{Th} = \frac{V_o}{i_o} = \frac{1\ V}{2.25\ A} = \boxed{0.44\ \Omega}$

❖ To find $V_{Th}$, use Mesh analysis on the original circuit as shown in figure below,

For super mesh (1) & (2),

$\boxed{I_2 = I_1 + 1.5I_x}$ …(6)

& $\boxed{I_x = I_2}$ …(7)

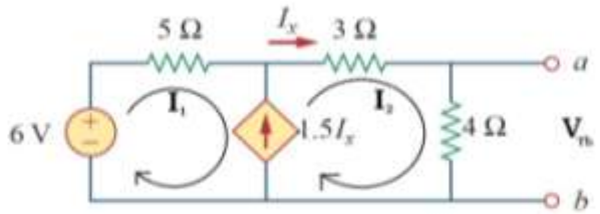



Figure (7.8)

Substitute equation (7) in (6), we get,

$\boxed{I_1 = -0.5I_2}$ …(8)

$-6 + 5I_1 + 3I_2 + 4I_2 = 0$

$\therefore$ $\boxed{5I_1 + 7I_2 = 6}$ …(9)

Substitute equation (8) in (9), we get,

$\boxed{I_2 = 1.33\ A}$

$\boxed{V_{Th} = 4 \times I_2 = 4 \times 1.33 = 5.33\ V}$

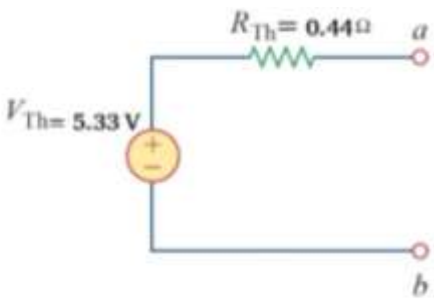



Figure (7.9)

**H.W.:-** Solve example (2) by using Nodal analysis.

**Example 3:** Find the current flowing through the 4 Ω resistor of **Figure (7.10)** using Thevenin theorem, if (i) E= 2V and (ii) E = 12V..

**Solution:**

When we remove E and 4 Ω resistor, the circuit becomes as shown in **Figure (7.11).**

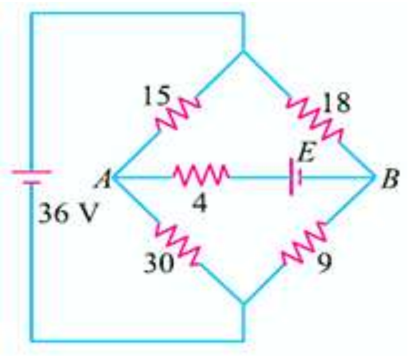



**Figure (7.10)**

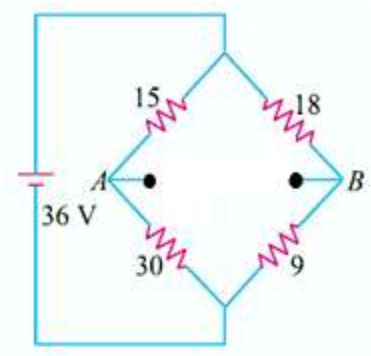



**Figure (7.11)**

- For finding $R_{th}$ i.e. the circuit resistance as viewed from terminals A and B, the battery has been short-circuited as shown in **Figure (7.12),** we can redraw the circuit as shown in **Figure (7.13).**

  It is seen from **Figure (7.13)** that, $R_{th} = R_{AB} = 15 \parallel 30 + 18 \parallel 9 = 16\ \Omega$.

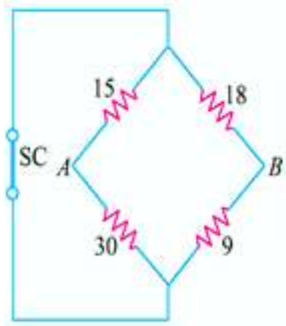



**Figure (7.12)**

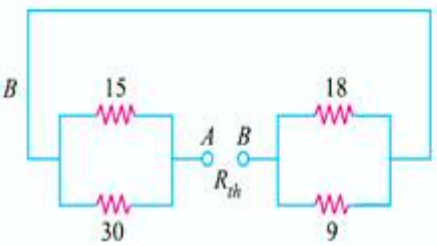



**Figure (7.13)**

- We will find $V_{th} = V_{AB}$ with the help of **Figure (7.11)** and redraw it as shown in **Figure (7.14).** To find voltages for each resistance we use voltage divider rule,

  $v_1 = 36 \times \frac{15}{15+30} = 12V$

  $v_2 = 36 \times \frac{18}{18+9} = 24V$

  Now apply KVL,

  $V_{Th} + v_1 - v_2 = 0$

  $\therefore V_{Th} = v_2 - v_1 = 24 - 12 = 12\ V$

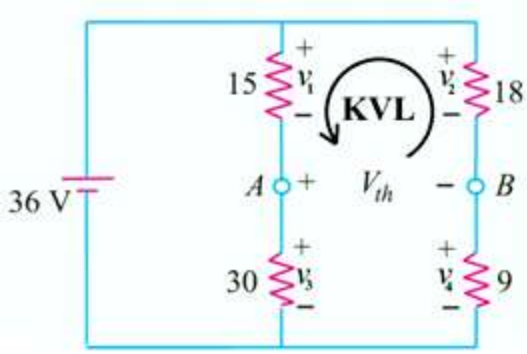



**Figure (7.14)**

**Or ,**

$$-V_{Th} + v_3 - v_4 = 0$$

$$\therefore V_{Th} = v_3 - v_4$$

$$v_3 = 36 \times \frac{30}{15+30} = 24V$$

$$v_4 = 36 \times \frac{9}{18+9} = 12V$$

$$\therefore V_{Th} = v_3 - v_4 = 24 - 12 = \mathbf{12\ V}$$

❖ The Thevenin's equivalent circuit is shown in **Figure (7.15)**.
The current through 4 Ω resistor is ,

(i) $$I = \frac{V_{Th} - E}{R_{Th} + 4} = \frac{12 - 2}{16 + 4} = \mathbf{0.5\ A}$$

(ii) $$I = \frac{V_{Th} - E}{R_{Th} + 4} = \frac{12 - 12}{16 + 4} = \mathbf{0\ A}$$

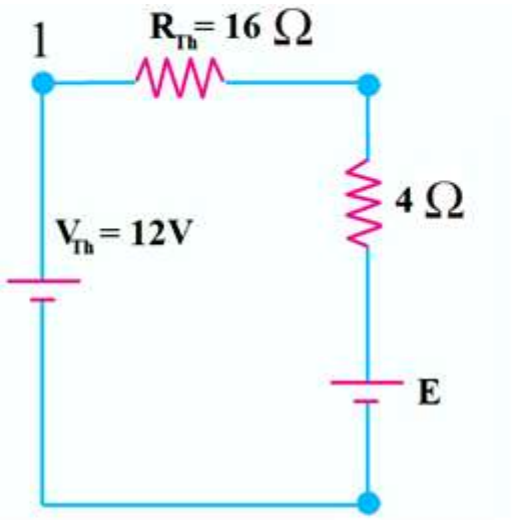



**Figure (7.15)**

## Special Case: Circuit have dependent sources only (without independent sources)

When there exist dependent sources only & there are not any independent source, then we must excite the circuit externally. The simplest approach is to excite the circuit with either a 1-V voltage source or a 1-A current source. Since we will end up with an equivalent resistance (either positive or negative).

In addition we will not have a value for Thevenin's equivalent voltage ($V_{Th} = 0$); only $R_{Th}$ have a value which may be (either positive or negative).

**Example 3:** Determine the Thevenin equivalent of the circuit in **Figure (7.16)** at terminals a-b.

**Solution:**

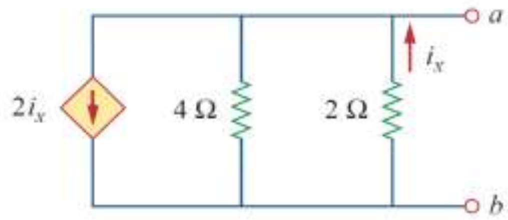



**Figure (7.16)**

To find $R_{Th}$, apply a independent current source of 1A across terminals a & b as shown in **Figure (7.17)**.

Use nodal analysis to find the voltage across current source,

$$2i_x + \frac{v_o}{4} + \frac{v_o}{2} - 1 = 0 \qquad \ldots(1)$$

$$\& \; i_x = \frac{0 - v_o}{2} = \frac{-v_o}{2} \qquad \ldots(2)$$

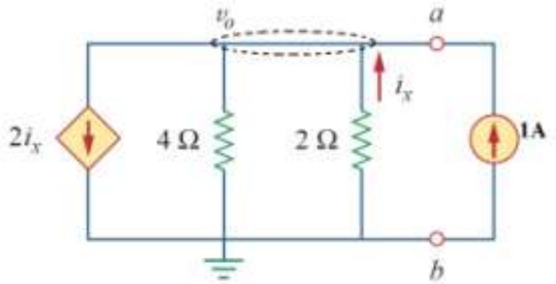



**Figure (7.16)**

Substitute Equ. (2) in (1) we get,

$$2\left(\frac{-v_o}{2}\right) + \frac{v_o}{4} + \frac{v_o}{2} - 1 = 0$$

$$\therefore v_o = \mathbf{-4V} \quad \Rightarrow \quad \therefore R_{Th} = \frac{v_o}{1} = \mathbf{-4\ \Omega}$$

The negative value of the resistance tells us that, according to the passive sign convention, the circuit in **Figure (7.16)** is supplying power. Of course, the resistors in **Figure (7.16)** cannot supply power (they absorb power); it is the dependent source that supplies the power. This is an example of how a dependent source and resistors could be used to simulate negative resistance.

We know this is not possible in a passive circuit, but in this circuit we do have an active device (the dependent current source). Thus, the equivalent circuit is essentially an active circuit that can supply power.

**Note** that as there is no independent source in the original circuit, then $V_{Th} = \mathbf{0\ V}$.

Lecture (8)

# Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source $I_N$ in parallel with a resistor $R_N$, where $I_N$ is the short-circuit current through the terminals and $R_N$ is the input or equivalent resistance at the terminals when the independent sources are turned off.

## How to find Norton's equivalent circuit

This procedure is based on the first statement of the theorem given above.

**1.** Remove the resistance (if any) across the two given terminals and put a short circuit across them.

**2.** Compute the short-circuit current $i_{sc}$. $I_N = i_{sc}$

**3.** Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open-circuits.

**4.** Next, find the resistance $R_N$ of the network as looked into from the given terminals. It is exactly the same as $R_{Th}$.

**Note:** the procedure for finding Norton's equivalent resistance is the same as the as the procedure for finding Thevenin's equivalent resistance, so $R_N = R_{Th}$

**5.** The current source ($i_{sc}$) joined in parallel across $R_N$ between the two terminals gives Norton's equivalent circuit.

Thus, the circuit in **Figure (8.1)(a)** can be replaced by the one in **Figure (8.1)(b).**

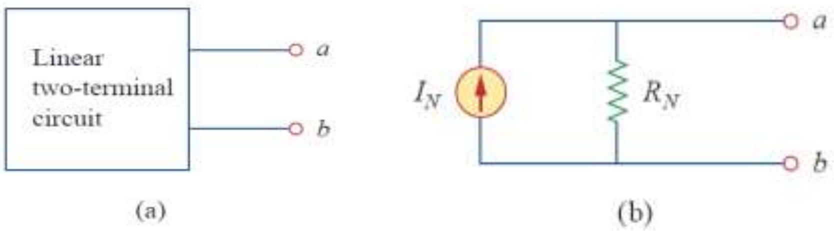



**Figure (8.1):** (a) Original circuit, (b) Norton equivalent circuit.

Observe the close relationship between Norton's and Thevenin's theorems:

$R_N = R_{Th}$ and $I_N = \frac{V_{Th}}{R_{Th}}$

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

$V_{Th} = v_{oc}$ , $I_N = i_{sc}$ , $R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$

**Example 1:** Using Norton's theorem to solve (Example (1) of Thevenin's Theorem).

**Solution:**

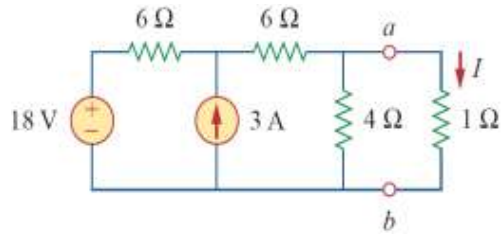



**Figure (8.2)**

- We find $R_N$ in the same way as we find $R_{Th}$, so, $R_N = R_{Th} = 3\ \Omega$

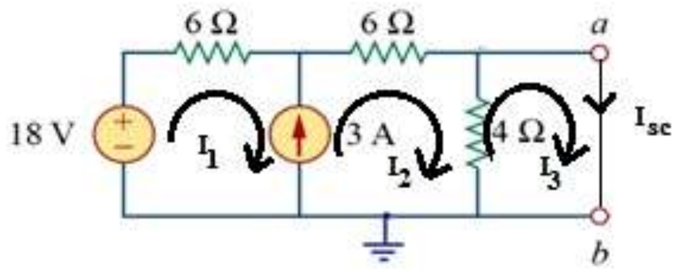



**Figure (8.3)**

- To find ($I_N$) we must short the terminals a & b to find the sort circuit current $I_{sc}$,
  Where $I_N = I_{sc}$

  **Use Mesh analysis,**

  For super mesh (1) & (2),

  $I_1 = I_2 - 3$ …(1)

  $-18 + 6I_1 + 6I_2 + 4(I_2 - I_3) = 0$

  $3I_1 + 5I_2 - 2I_3 = 9$ …(2)

  Substitutes Equ.s (1) in (2), we get,

  $4I_2 - I_3 = 9$ …(3)

For mesh (3),

$4(I_3 - I_2) = 0$

$\therefore \boxed{I_2 = I_3}$ …(4)

Substitute Equ. (4) in (3), we get,

$I_3 = 3A = I_{sc} = I_N$

$I_L = I_N \times \frac{R_N}{R_N + R_L} = 3 \times \frac{3}{3+1} = 2.25\ A$

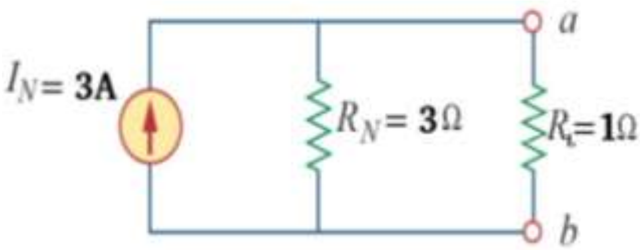



Figure (8.4)

Also short circuit current can be found by,

$I_N = \frac{V_{Th}}{R_{Th}} = \frac{9}{3} = 3A$

**H.W./** Solve example (1) by using Nodal analysis.

**Example 2:** Find Norton's equivalent circuit of (Example (2) of Thevenin's Theorem).

**Solution:**

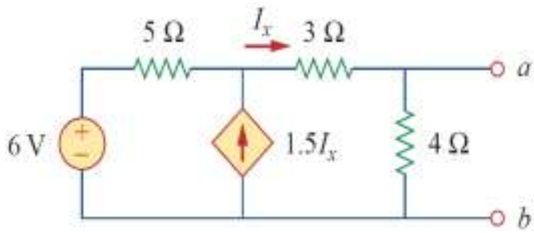



Figure (8.5)

❖ We find $R_N$ in the same way as we find $R_{Th}$, so,

$R_N = R_{Th} = 0.44\ \Omega$

- To find ($I_N$) we must short the terminals a & b to find the sort circuit current $I_{sc}$, Where $\boxed{I_N = I_{sc}}$

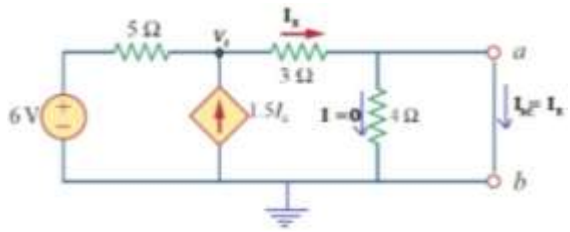



Figure (8.6)

- The S.C. remove the parallel resistances with it, i.e. (4Ω resistor) because the current will not flow in the (4Ω resistor) but tends to flow in the S.C. path ( because it has resistance of zero Ω).

Equivalent resistance of (4Ω resistor) & (S.C. resistor) is,

$$R_{eq} = \frac{4\times R_{sc}}{4+R_{sc}} = \frac{4\times 0}{4+0} = 0\ \Omega = R_{sc}$$

The circuit can be redrawn as shown in figure.

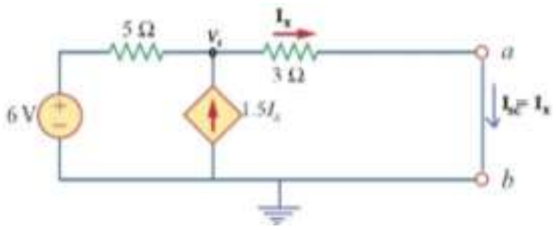



Figure (8.7)

**Use Nodal analysis to find $I_{sc}$,** $\frac{V_1 - 6}{5} - 1.5\,I_x + \frac{V_1}{3} = 0$

$\therefore \boxed{8V_1 - 22.5\,I_x = 18}$ …(1)

It is clearly from figure that,

$\frac{V_1}{3} = I_x \Longrightarrow \boxed{V_1 = 3\,I_x}$ …(2)

Substitute Equ. (2) in (1) we get,

$\boxed{I_x = 12\ A = I_{sc} = I_N}$

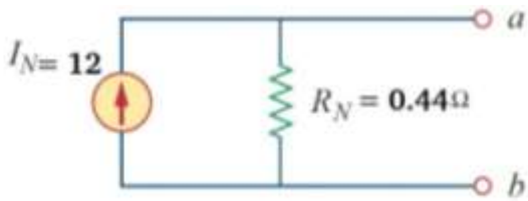



Figure (8.8)

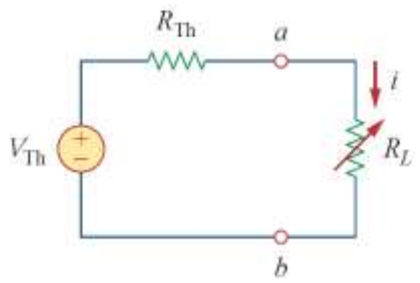
**H.W./** Solve example (2) by using Mesh analysis.

**Lecture (9)**

# Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance $R_L$ If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Figure (1), the power delivered to the load is




$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 \times R_L \quad \ldots(1)$$

**Figure (9.1):** The circuit used for maximum power transfer.

**Maximum power** is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate $p$ in equation (1) with respect to $R_L$ and set the result equal to zero. We obtain,

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$= V_{Th}^2 \left[\frac{R_{Th} + R_L - 2R_L}{(R_{Th} + R_L)^3}\right] = 0$$

This implies that

$0 = (R_{Th} + R_L - 2R_L) = (R_{Th} - R_L)$

This yields

$$R_L = R_{Th} \quad \ldots(2)$$

The maximum power transferred is obtained by substituting Eq. (2) into Eq. (1), for

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} \quad \ldots(3)$$

**Note:** Equation (3) applies only when $R_L = R_{Th}$. When $R_L \neq R_{Th}$, we compute the power delivered to the load using Equation (1).

**Example 1:** Find the value of $R_L$ such that maximum possible power will be transferred to $R_L$. Find also the value of the maximum power and the power supplied by source under these conditions.

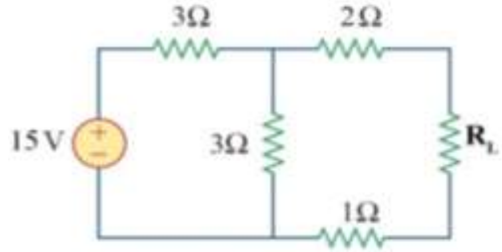



Figure (9.2)

**Solution:**

For maximum power transferred, $\boxed{R_L = R_{Th}}$

To find $R_{Th}$, remove $R_L$ & short the independent voltage source.

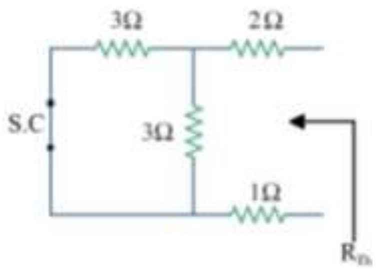



Figure (9.3)

$$\therefore R_{Th} = [3 \| 3] + 2 + 1 = \left[\frac{3\times3}{3+3}\right] + 2 + 1 = 4.5\,\Omega$$

$$\boxed{R_L = R_{Th} = 4.5\,\Omega}$$

Now for finding $V_{Th}$ note that in this circuit,

$V_{Th}$ = voltage across (3Ω) resistor because there is no current flows in (2Ω) & (1Ω) resistors because these resistors are open.

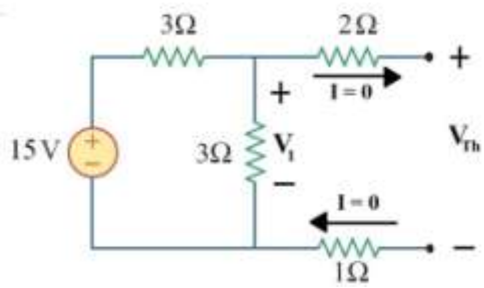



Figure (9.4)

$$\therefore V_{Th} = V_1 = 15 \times \left[\frac{3}{3+3}\right] = \boxed{7.5\text{ V}}$$ , [voltage divider rule]

$$P_{max.} = \frac{V_{Th}^2}{4R_{Th}} = \frac{7.5^2}{4 \times 4.5} = \boxed{3.125\text{ W}}$$

Since the same power is developed in $R_{Th}$, so power supplied by the source is $= 2 \times 3.125 = \boxed{6.250\text{ W}}$

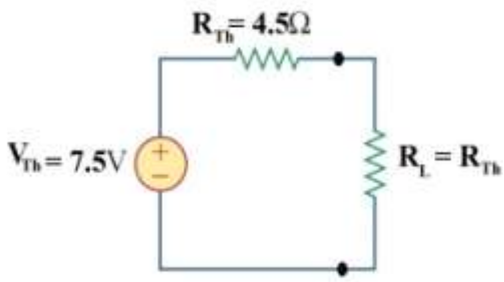



Figure (9.5)

**Example 2:** Determine the value of $R_L$ that will draw maximum power from the rest of the circuit shown. Also calculate the maximum power.

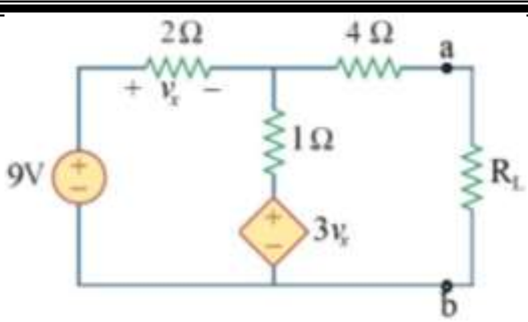



**Figure (9.6)**

**Solution:**

For maximum power transferred, $\boxed{R_L = R_{Th}}$

& $\boxed{P_{max} = \frac{V_{Th}^{\ 2}}{4R_{Th}}}$

So we must find $R_{Th}$ & $V_{Th}$.